

③ If  $\lim_{x \rightarrow c} f(x) = L$ ,  $\lim_{x \rightarrow c} g(x) = M$ ,  $\lim_{x \rightarrow c} h(x) = N$ ,

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then  $\lim_{x \rightarrow c} (3f(x) + 4g(x) - 5h(x)) = 3L + 4M - 5N$

Given  $\epsilon > 0$   $\exists \delta_1, \delta_2, \delta_3$  s.t.

$$0 < |x - c| < \delta_1 \Rightarrow -\frac{\epsilon}{12} < f(x) - L < \frac{\epsilon}{12}$$

$$0 < |x - c| < \delta_2 \Rightarrow -\frac{\epsilon}{12} < g(x) - M < \frac{\epsilon}{12}$$

$$0 < |x - c| < \delta_3 \Rightarrow -\frac{\epsilon}{12} < N - h(x) < \frac{\epsilon}{12}$$

take  $\delta = \min\{\delta_1, \delta_2, \delta_3\}$

for  $0 < |x - c| < \delta$   $-\frac{3}{12}\epsilon < 3f(x) - 3L < \frac{3}{12}\epsilon$

$$-\frac{4}{12}\epsilon < 4g(x) - 4M < \frac{4}{12}\epsilon$$

$$+ \quad -\frac{5}{12}\epsilon < 5N - 5h(x) < \frac{5}{12}\epsilon$$

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$$-\epsilon < 3f(x) + 4g(x) - 5h(x) - (3L + 4M - 5N) < \epsilon$$

得證  $\lim_{x \rightarrow c} (3f(x) + 4g(x) - 5h(x)) = 3L + 4M - 5N$