$x \to 3$

-25 3. (30 pts) Use the $\varepsilon - \delta$ argument to prove that:

If $\lim_{x \to c} f(x) = L$, $\lim_{x \to c} g(x) = M$ and $\lim_{x \to c} h(x) = N$, then

$$\lim_{x \to c} \Big(3f(x) + 4g(x) - 5h(x)\Big) = 3L + 4M - 5N$$

take $\delta = \min\{\delta_1, \delta_2, \delta_3\}$

for $0 < |x-c| < \delta$ -20

3. Given $\varepsilon > 0$, there exists $\delta_1 > 0, \delta_2 > 0, \delta_3 > 0$, then

$$\begin{cases} 0 < |x-c| < \delta_1 \Rightarrow |f(x) - L| < \frac{\varepsilon}{2\varepsilon} \\ 0 < |x-c| < \delta_2 \Rightarrow |g(x) - M| < \frac{\varepsilon}{2} \\ 0 < |x-c| < \delta_3 \Rightarrow |h(x) - N| < \frac{\varepsilon}{2} \end{cases} \Rightarrow \begin{cases} -\frac{3}{2}\varepsilon < 3f(x) - 3L < \frac{3}{2}\varepsilon \\ -2\varepsilon < 4g(x) - 4M < 2\varepsilon \\ \frac{5}{2}\varepsilon > -5h(x) + 5N > -\frac{5}{2}\varepsilon \end{cases} \Rightarrow \Big|\big(3f(x) + 4g(x) - 5h(x)\big) - (3L + 4M - 5N)\Big| < \varepsilon$$

why? -5