

Def: For any  $\epsilon > 0$ , there exists a  $\delta > 0$  such that

$$0 < |x - c| < \delta \Rightarrow |f(x) - L| < \epsilon$$

<方法>  $\rightarrow$  Given  $\epsilon > 0$ ,  $|(x-1)^2 - 4| < \epsilon$

$$\Leftrightarrow 4 - \epsilon < (x-1)^2 < 4 + \epsilon$$

(if  $\epsilon < 4$ )  $\rightarrow$

$$\Leftrightarrow \sqrt{4 - \epsilon} < x - 1 < \sqrt{4 + \epsilon}$$

$$\Leftrightarrow -2 + \sqrt{4 - \epsilon} < x - 3 < -2 + \sqrt{4 + \epsilon}$$

We hope  $\begin{cases} \delta \geq -2 + \sqrt{4 - \epsilon} \\ \delta \leq -2 + \sqrt{4 + \epsilon} \end{cases}$

So, if  $0 < \epsilon < 4$ ,

then take  $\delta = \min \{ 2 - \sqrt{4 - \epsilon}, -2 + \sqrt{4 + \epsilon} \}$

$$\text{Thus, } 0 < |x - 3| < \delta \Rightarrow |(x-1)^2 - 4| < \epsilon$$

<方法>  $\Rightarrow$  Given  $\epsilon > 0$ , there exists a  $\delta = \min \{ 1, \frac{\epsilon}{5} \}$

if  $0 < |x - 3| < \delta$  then since  $|x - 3| < \delta \leq 1$

$$\Rightarrow |x + 1| \leq |x - 3| + 4 < 5$$

$$\text{Thus, } |(x-1)^2 - 4| = |x+1||x-3| < 5|x-3| < 5\delta \leq \epsilon$$