2. Def: For all $\varepsilon>0$, there exists $\delta>0$ s.t.
if $0<|x-c|<\delta$ then $|f(x)-L|<\varepsilon$

<方法一>
Given $\varepsilon>0$, $|(x-1)^2-4|<\varepsilon$

$\Leftrightarrow 4-\varepsilon<(x-1)^2<4+\varepsilon$

(if $\varepsilon<4$) $\Leftrightarrow \sqrt{4-\varepsilon}<x-1<\sqrt{4+\varepsilon}$

$\Leftrightarrow -2+\sqrt{4-\varepsilon}<x-3<-2+\sqrt{4+\varepsilon}$

We hope $\begin{cases} -\delta \geq -2+\sqrt{4-\varepsilon} \\ \delta \leq -2+\sqrt{4+\varepsilon} \end{cases}$, thus take $\delta=\min\left\{2-\sqrt{4-\varepsilon}, -2+\sqrt{4+\varepsilon}\right\}$

Therefore, given $0<\varepsilon<4$, take $\delta=\min\left\{2-\sqrt{4-\varepsilon}, -2+\sqrt{4+\varepsilon}\right\}$
we have $0<|x-3|<\delta \Rightarrow |(x-1)^2-4|<\varepsilon$

<方法二>
Given $\varepsilon>0$, there exists $\delta=\min\left\{1, \frac{\varepsilon}{5}\right\}$
if $0<|x-3|<\delta$ then since $|x-3|<\delta\leq 1$
$\Rightarrow |x+1|\leq |x-3|+4<5$
Thus, $|(x-1)^2-4|=|x+1||x-3|<5|x-3|<5\delta\leq\varepsilon$

# 常犯錯誤

(1) take $\delta = \min\{2-\sqrt{-\varepsilon+4}, \sqrt{\varepsilon+4}-2\}$ without saying $0<\varepsilon<4$

(2) $\delta$ depends on $c$ and $\varepsilon$

if you take $\delta = \frac{\varepsilon}{x+1}$, then you're wrong

(3) 很多人寫 $\delta = \sqrt{\varepsilon+4}-2$ (When $0<\varepsilon<4$, $\sqrt{\varepsilon+4}-2 < 2-\sqrt{-\varepsilon+4}$)

很不巧它是對的，無法扣分