Brief solutions to Quiz 9

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1. Solve for $y' + (\tan x)y = \cos x$, $y(0) = 1$.

   Ans:

   Multiply the equation by $\exp(\int \tan x\, dx) = \frac{1}{|\cos x|} = \pm\frac{1}{\cos x}$.

   $\frac{1}{\cos x}y' + \frac{\sin x}{\cos^2 x}y = 1$,

   $(\frac{y}{\cos x})' = 1$,

   $\frac{y}{\cos x} = x + C$, $y(0) = 1$, therefore $C = 1$.

   $y = (x+1)\cos x$.

2. Evaluate $\int_1^2 \frac{\cosh(\ln t)}{t} dt$.

   Ans:

   $= \int_1^2 \cosh(\ln t) d(\ln t) = \sinh(\ln t)\Big|_1^2 = \sinh(\ln 2)(= \frac{3}{4})$.

3. Order the functions n, $\sqrt{n}\log_2 n$, $(\log_2 n)^5$ by rate of growth as $n \to \infty$. You can use the notations $a \ll b$ to say b grows faster than a, and $a \approx b$ to say a and b grow at the same rate.

   Ans:

   Use L'Hôpital's Rule to find $\lim_{n\to\infty} \frac{\ln n}{n^{\frac{1}{8}}} = 0 \implies \lim_{n\to\infty} \frac{(\ln n)^4}{n^{\frac{1}{2}}} = 0 \implies \sqrt{n} \ll (\log_2 n)^4$.

   Therefore $(\log_2 n)^5 \ll \sqrt{n}\log_2 n$.

   Similarly, from L'Hôpital's Rule $\lim_{n\to\infty} \frac{\ln n}{n^{\frac{1}{2}}} = 0$, therefore $\sqrt{n}\log_2 n \ll n$.

   Answer: $(\log_2 n)^5 \ll \sqrt{n}\log_2 n \ll n$.