Brief solutions to Quiz 8

Dec 21, 2021

1. Find the length of the curve $y = f(x) = \int_0^x \sqrt{\cos t}\, dt$ from $x = 0$ to $x = \frac{\pi}{2}$.

   Ans:

   $f'(x) = \sqrt{\cos x}$. $ds = \sqrt{1 + (f'(x))^2}dx$.

   $L = \int_0^{\frac{\pi}{2}} ds = \int_0^{\frac{\pi}{2}} \sqrt{1+(f'(x))^2}dx = \int_0^{\frac{\pi}{2}} \sqrt{1+\cos x}\, dx = \int_0^{\frac{\pi}{2}} \sqrt{2\cos^2 \frac{x}{2}}\, dx.$

   $= \int_0^{\frac{\pi}{2}} \sqrt{2}\cos\frac{x}{2}\, dx = 2\sqrt{2}\sin\frac{x}{2}\Big|_0^{\frac{\pi}{2}} = 2.$

2. A torus (donut) is generated by revolving the circle $(x-2)^2 + y^2 = 1$ around the y axis. Find the surface area of the torus. Give all details.

   Ans:

   See page 6 of Lecture 22 note and page 1-2 of Lecture 23 note.

3. Find the solutions for $\frac{dy}{dx} = 3x^2e^{-y}$ on $\{x \geq 0\}$ with $y(0) = 0$.

   Ans:

   $\frac{dy}{dx} = 3x^2e^{-y}$,

   $e^y dy = 3x^2 dx$,

   $\int e^y dy = \int 3x^2 dx$,

   $e^y = x^3 + C$,

   $y = \ln(x^3 + C)$, $y(0) = 0$, therefore $C = 1$.

   Answer: $y = \ln(x^3 + 1)$.