Brief solutions to Quiz 7

Dec 21, 2021

1. Find the volume of the region bounded by $z \geq 0$, $z \leq x$ and $x^2 + y^2 \leq 9$ (i.e. above the xy plane, below the plane $x = z$ and inside the cylinder $x^2 + y^2 = 9$).

   Ans:

   See page 2-3 of Lecture 21 note.

2. Use two different methods (disks/washers and cylindrical shells) to find the volume of revolution obtained by rotating the region $\{0 \leq x \leq 2,\ x^2 \leq y \leq 2x\}$ around the x-axis.

   Ans:

   Method of disks(washers):

$$V = \int_0^2 \pi\big((2x)^2 - (x^2)^2\big)dx = \pi\big(\frac{4}{3}x^3 - \frac{1}{5}x^5\big)\Big|_0^2 = \frac{64}{15}\pi$$

   Method of cylindrical shells:

$$V = \int_0^4 2\pi y\big(\sqrt{y} - \frac{y}{2}\big)dy = 2\pi\big(\frac{2}{5}y^{\frac{5}{2}} - \frac{1}{6}y^3\big)\Big|_0^4 = \frac{64}{15}\pi$$

3. Same as above, but change the rotation axis to y- axis.

   Ans:

   Method of disks(washers):

$$V = \int_0^4 \pi\big((\sqrt{y})^2 - (\frac{y}{2})^2\big)dy = \pi\big(\frac{1}{2}y^2 - \frac{1}{12}y^3\big)\Big|_0^4 = \frac{8}{3}\pi$$

   Method of cylindrical shells:

$$V = \int_0^2 2\pi x\big(2x - x^2\big)dx = 2\pi\big(\frac{2}{3}x^3 - \frac{1}{4}x^4\big)\Big|_0^2 = \frac{8}{3}\pi$$