Brief solutions to Quiz 6

Nov 30, 2021

1. (36 pts) Sketch the graph of $f(x) = x^4 - 4x^3 + 10$ (For your reference, $f'(x) = 4x^2(x-3)$, $f''(x) = 12x(x-2)$) on $[-1, 4]$. Identify all local extremes and points of inflection, if any. Note that the domain is a finite closed interval.

   Ans:

   See page 3-5 of Lecture 15 note. Additional local maxima: $f(-1)$ and $f(4)$.

2. (16 pts) Evaluate $\lim\limits_{x \to 0} \dfrac{1}{x} - \dfrac{1}{\sin x}$.

   Ans: See Example 6 of Section 4.5.

3. (16 pts) Evaluate $\lim\limits_{x \to \infty} \left(\dfrac{x+1}{x-1} \right)^x$.

   Ans:

   $$= \lim_{x \to \infty} \left(1 + \frac{2}{x-1} \right)^x = e^{\left(\lim\limits_{x \to \infty} x \ln(1 + \frac{2}{x-1}) \right)}$$

   $$\lim_{x \to \infty} x \ln(1 + \frac{2}{x-1}) = \lim_{x \to \infty} \frac{\ln(1 + \frac{2}{x-1})}{\frac{1}{x}} = \lim_{x \to \infty} \frac{\frac{-\frac{2}{(x-1)^2}}{1 + \frac{2}{x-1}}}{\frac{-1}{x^2}} = 2$$

   Answer $= e^2$.

4. (16 pts) Evaluate $\lim\limits_{x \to 0^+} x^x$.

   Ans: $= e^{\left(\lim\limits_{x \to 0^+} x \ln x \right)}$

   $$\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x \to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = 0$$

   Answer $= e^0 = 1$.

5. (16 pts) Evaluate $\lim\limits_{x \to 0^+} \dfrac{e^{\frac{-1}{x}}}{x}$.

   Ans:

   See page 6 of Lecture 16 note.