Brief solutions to Quiz 5

Nov 23, 2021

1. (30 pts) Find absolute minimum and absolute maximum of $f(x) = \dfrac{1}{x} + \ln x$ on the interval $[0.5, 4]$. For your reference, $\ln 2 \approx 0.6931$.

   Ans:

   $f'(x) = \dfrac{-1}{x^2} + \dfrac{1}{x} = \dfrac{x-1}{x^2}$.

   Critical points in $(0.5, 4)$: $x = 1$ only.

   Compare values of $f(0.5)$, $f(1)$ and $f(4)$ shows that $f(1) =$ absolute minimum, $f(4) =$ absolute maximum.

2. (20+20 pts) Suppose that f'' is continuous on $[a, b]$ and f has 3 distinct zeros on $[a, b]$. Show that f'' has at least one zero in (a, b). First state clearly the content of theorem you use (need not prove the theorem).

   Ans:

   Rolle's Theorem or Mean Value Theorem (both applicable): see textbook.

   Apply Rolle's Theorem to f to get two distinct zeros of f' (you fill in the details). Then apply Rolle's Theorem again to f' to get a zero of f'' (you fill in the details again).

3. (30 pts) Find all critical points of $f(x) = x^{\frac{2}{3}}(x - 4)$. For each one of them, use first derivative test to determine whether it corresponds to a local minimum, a local maximum or neither.

   Ans:

   See page 9 of Lecture 14 note.

4. Common mistakes in problem 3:

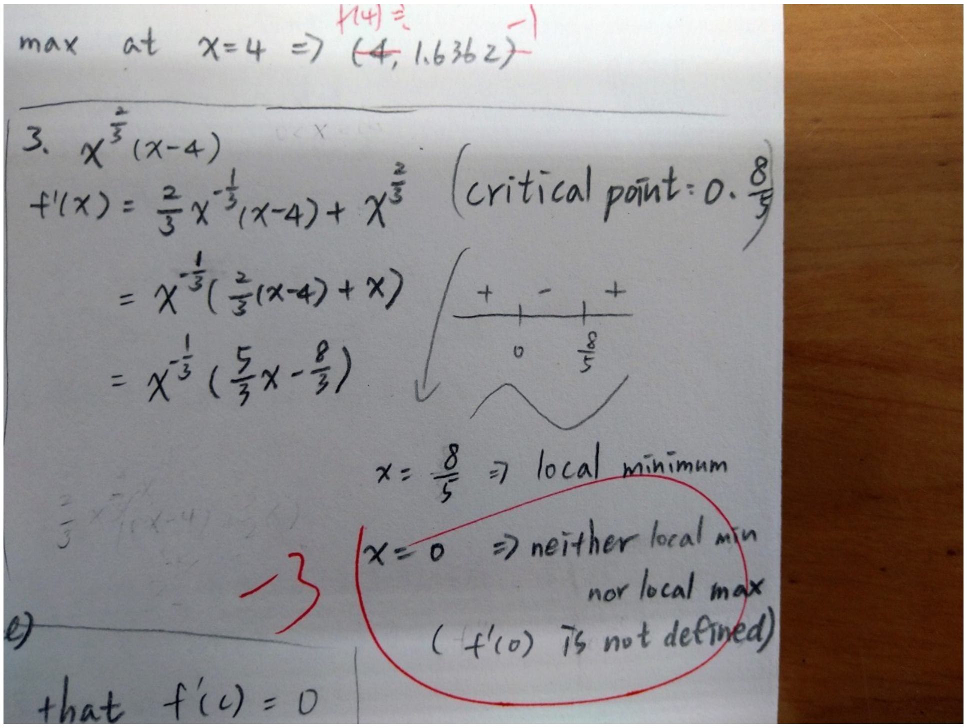


Figure 1: Common mistakes to problem 3-1

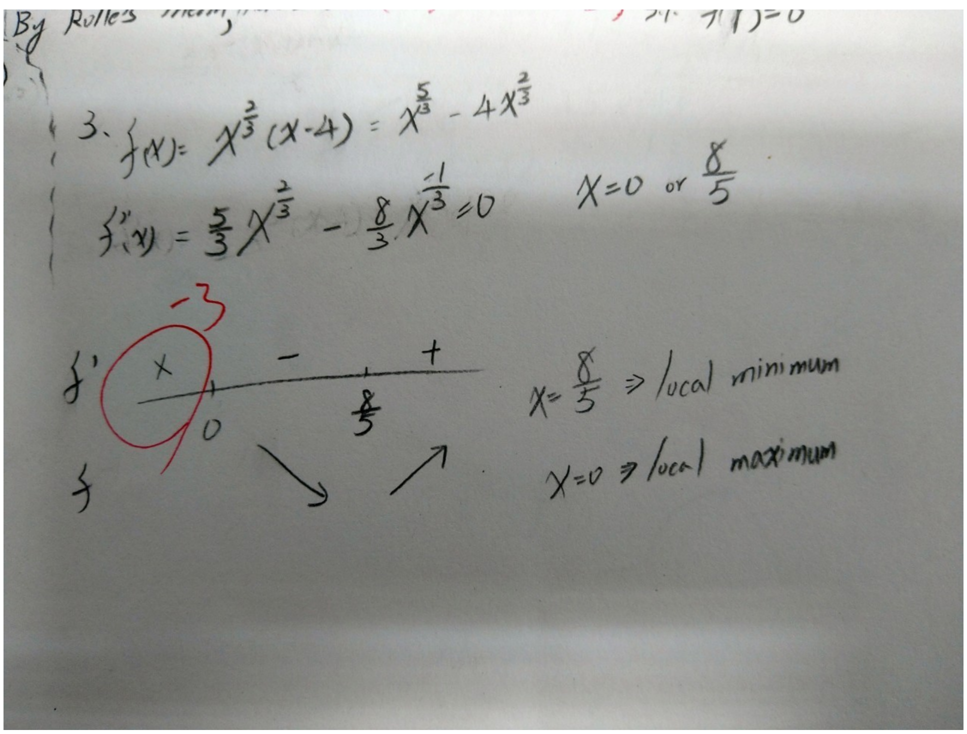
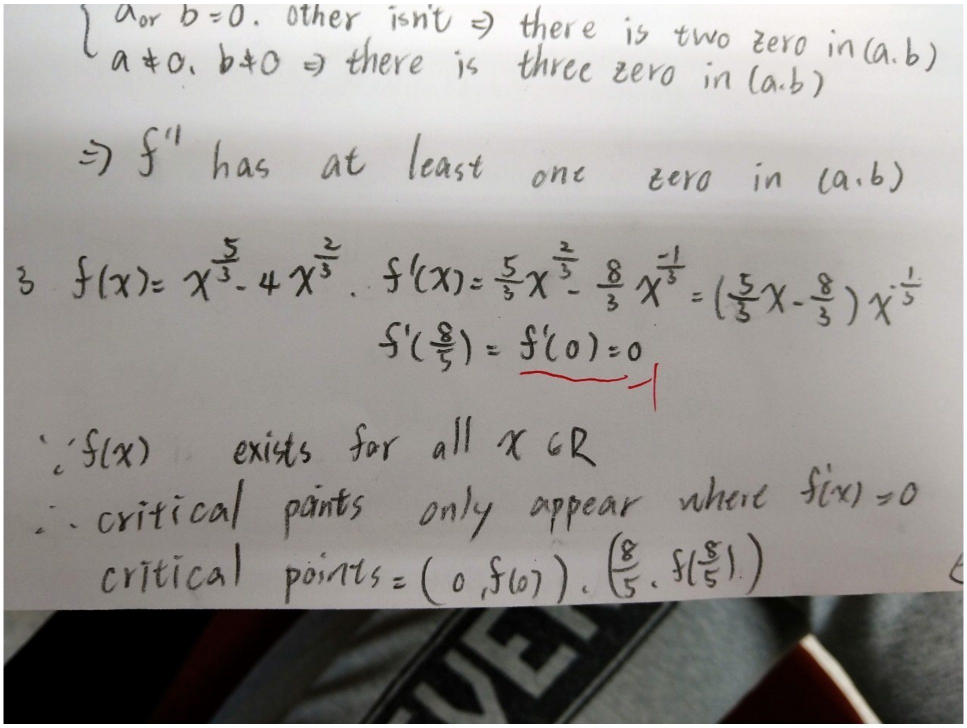


Figure 2: Common mistakes to problem 3-2



$\begin{cases} a \text{ or } b = 0. \text{ other isn't} \Rightarrow \text{there is two zero in } (a.b) \\ a \neq 0,\ b \neq 0 \Rightarrow \text{there is three zero in } (a.b) \end{cases}$

$\Rightarrow f''$ has at least one zero in $(a.b)$

3 $f(x) = x^{\frac{5}{3}} - 4x^{\frac{2}{3}}$. $f'(x) = \frac{5}{3}x^{\frac{2}{3}} - \frac{8}{3}x^{\frac{-1}{3}} = (\frac{5}{3}x - \frac{8}{3})x^{-\frac{1}{3}}$

$f'(\frac{8}{5}) = f'(0) = 0$

$\because f(x)$ exists for all $x \in R$

$\therefore$ critical points only appear where $f'(x) = 0$

critical points $= (0, f(0)), (\frac{8}{5}, f(\frac{8}{5}))$

Figure 3: Common mistakes to problem 3-3