Brief solutions to Quiz 1

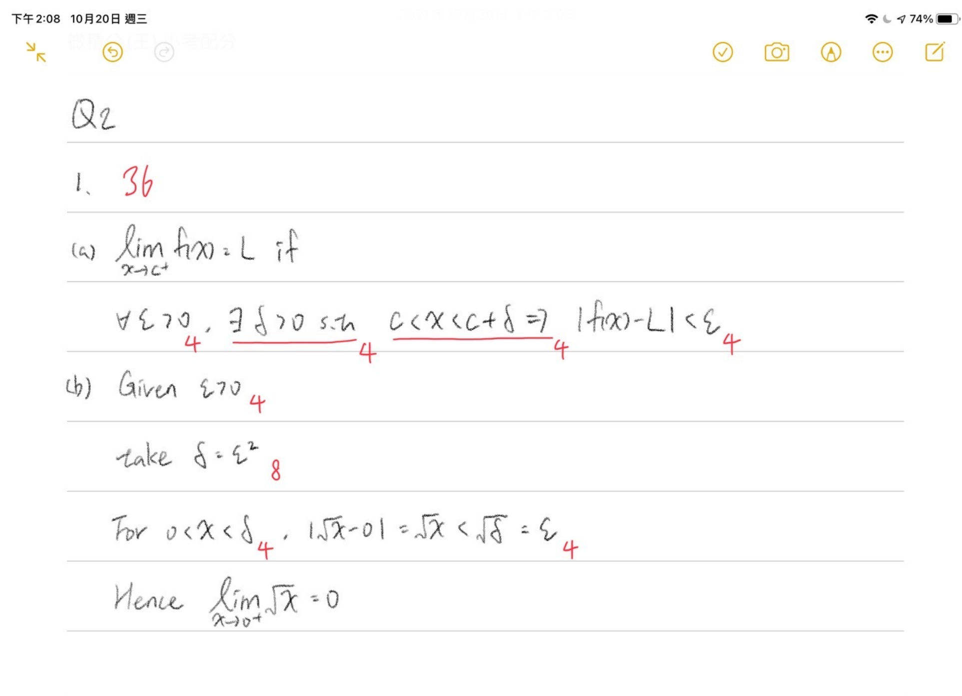
Oct 12, 2021

1. (36 pts)

   (a) State precise definition of $\lim_{x\to c^+} f(x) = L$.

   (b) Use it to prove that $\lim_{x\to 0^+} \sqrt{x} = 0$.

   Ans:



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Q2

1、 36

(a) $\lim_{x\to c^+} f(x) = L$ if

$\forall \varepsilon > 0$ 4, $\exists \delta > 0$ s.t. 4 $c < x < c+\delta \Rightarrow$ 4 $|f(x) - L| < \varepsilon$ 4

(b) Given $\varepsilon > 0$ 4

take $\delta = \varepsilon^2$ 8

For $0 < x < \delta$ 4, $|\sqrt{x} - 0| = \sqrt{x} < \sqrt{\delta} = \varepsilon$ 4

Hence $\lim_{x\to 0^+} \sqrt{x} = 0$

Figure 1: Answer to problem 1

2. (32 pts) Evaluate $\lim_{\theta\to 0} f(\theta)$ where $f(\theta) = \cos\left(\dfrac{\sin(1-\cos\theta)}{\tan^2\theta}\right)$. Take for granted that $\cos(\cdot)$ is a continuous function. State clearly any Theorem you use, but need not prove it. Note that $f(\theta)$ is NOT defined on $\theta = 0$.

   Remark: See also the hint in problem 3, homework week 04.

   Ans:

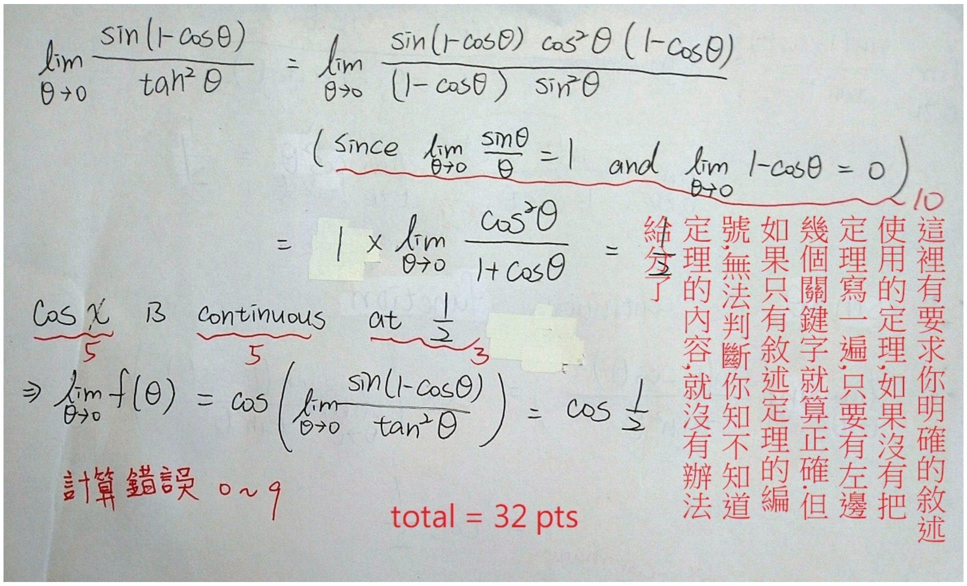


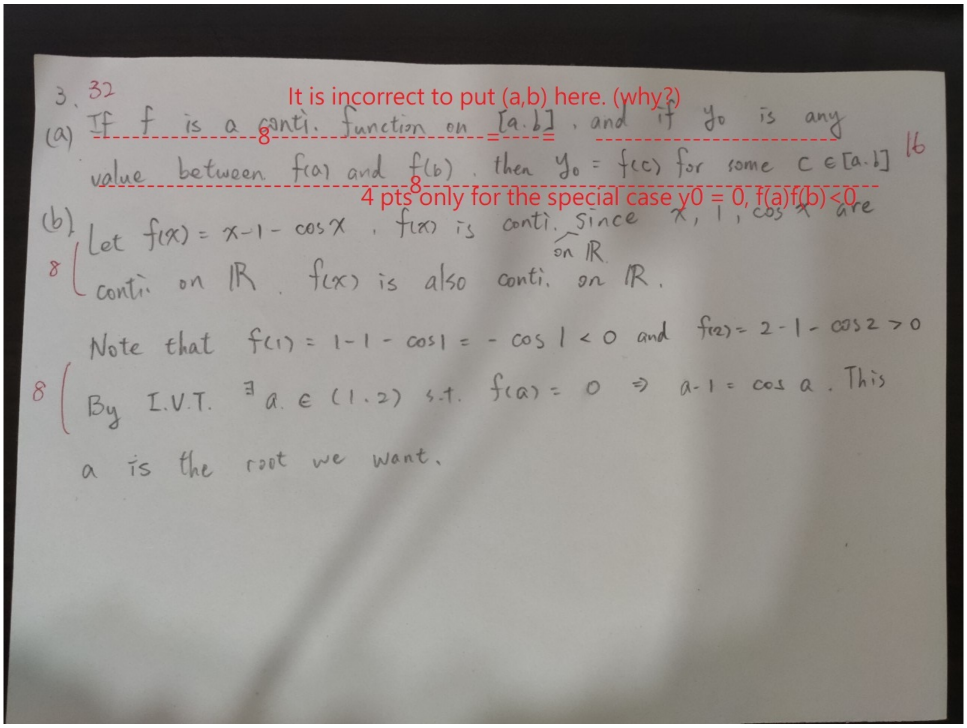
Figure 2: Answer to problem 2

3. (32 pts)

   (a) State the Intermediate Value Theorem.

   (b) Use it to locate a root of $x - 1 = \cos x$ on an interval of length 1. That is, find a c such that there is a root on $(c, c+1)$.

   Ans:



3. 32

It is incorrect to put (a,b) here. (why?)

(a) If f is a conti. function on $[a,b]$, and if y_0 is any value between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some $c \in [a,b]$ 16

8 8

4 pts only for the special case y0 = 0, f(a)f(b)<0

(b) 8 Let $f(x) = x - 1 - \cos x$, $f(x)$ is conti. on $\mathbb{R}$ since $x, 1, \cos x$ are conti. on $\mathbb{R}$. $f(x)$ is also conti. on $\mathbb{R}$.

Note that $f(1) = 1 - 1 - \cos 1 = -\cos 1 < 0$ and $f(2) = 2 - 1 - \cos 2 > 0$

8 By I.V.T. $\exists\, a \in (1,2)$ s.t. $f(a) = 0 \Rightarrow a - 1 = \cos a$. This a is the root we want.

Figure 3: Answer to problem 3

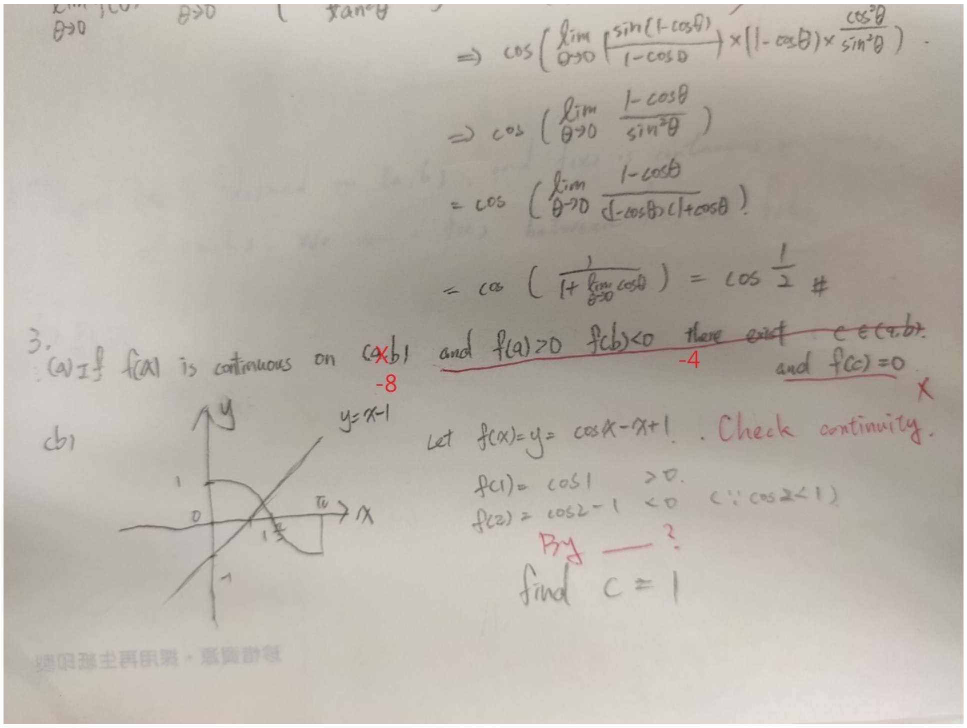


Figure 4: Some common mistakes to problem 3