Sec 6-3

25 $y(x) = \int_0^x \sqrt{\cos 2t}\, dt$ ⇒ $y'(x) = \sqrt{\cos 2x}$ by Fundamental thm of Caloulus

$\Rightarrow \int_0^{\frac{\pi}{4}} \sqrt{1+(y')^2}\, dx = \int_0^{\frac{\pi}{4}} \sqrt{1+\cos 2x}\, dx = \int_0^{\frac{\pi}{4}} \sqrt{2\cos^2 x}\, dx$

$\left(\because \cos x \geq 0 \text{ if } x \in [0, \frac{\pi}{4}]\right) = \sqrt{2} \int_0^{\frac{\pi}{4}} \cos x\, dx = \sqrt{2} \sin x \Big|_0^{\frac{\pi}{4}} = 1$

29. $9x^2 = y(y-3)^2 \Rightarrow 18x\,dx = [(y-3)^2 + 2y(y-3)]\,dy$

left

$\Rightarrow 6x\,dx = (y-3)(y-1)\,dy$

$\Rightarrow 36x^2 dx^2 = (y-3)^2(y-1)^2 dy^2$

$\Rightarrow dx^2 = \frac{(y-3)^2(y-1)^2}{4 \cdot y(y-3)^2}\, dy^2$

$ds^2 = dx^2 + dy^2$

$= \frac{(y-1)^2}{4y} dy^2 + dy^2 = \left[\frac{(y-1)^2 + 4y}{4y}\right] dy^2 = \frac{(y+1)^2}{4y} dy^2$

Sec 6.4

23. $\int 2\pi y\, ds = \int 2\pi y \sqrt{dx^2+dy^2} = \int 2\pi y \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\, dy$

$\Rightarrow S = \int_{y=1}^{y=2} 2\pi y \sqrt{(x')^2 + 1}\, dy$

$= \int_{y=1}^{y=2} 2\pi y \sqrt{\left(y^3 - \frac{1}{4y^3}\right)^2 + 1}\, dy \quad \left(x' = y^3 - \frac{1}{4y^3}\right)$

$= \int_1^2 2\pi y \sqrt{\left(y^3 + \frac{1}{4y^3}\right)^2}\, dy$

$= \int_1^2 2\pi y \left[y^3 + \frac{1}{4y^3}\right] dy \quad \left(\because y^3 + \frac{1}{4y^3} \geq 0 \text{ if } y \in [1,2]\right)$

$= 2\pi \left(\frac{y^5}{5} - \frac{1}{4y}\Big|_1^2\right) = \frac{253}{20}\pi$