

Sec 2.6

61. For any $M > 0$, take $\delta = \left(\frac{1}{M}\right)^{\frac{2}{3}} > 0$ s.t.

(a) if $0 < x < \delta$ then $f(x) \geq \frac{1}{x^{\frac{2}{3}}} > \frac{1}{\delta^{\frac{2}{3}}} = M$

(b) if $0 < -x < \delta$ then $f(x) \geq \frac{1}{x^{\frac{2}{3}}} > \frac{1}{\delta^{\frac{2}{3}}} = M$

(c) if $0 < x-1 < \delta$ then $f(x) \geq \frac{2}{(x-1)^{\frac{2}{3}}} > \frac{2}{\delta^{\frac{2}{3}}} = 2M > M$

(d) if $0 < 1-x < \delta$ then $f(x) \geq \frac{2}{(x-1)^{\frac{2}{3}}} > \frac{2}{\delta^{\frac{2}{3}}} = 2M > M$

85. $\lim_{x \rightarrow \infty} (\sqrt{x^2+3x} - \sqrt{x^2-2x}) = \lim_{x \rightarrow \infty} \frac{5x}{\sqrt{x^2+3x} + \sqrt{x^2-2x}}$

$$= \lim_{x \rightarrow \infty} \frac{5}{\sqrt{1+\frac{3}{x}} + \sqrt{1+\frac{2}{x}}}$$

$$\left(\lim_{x \rightarrow \infty} \frac{1}{x} = 0 \right) = \frac{5}{2}$$

92. For any $M > 0$, take $\delta = \sqrt{\frac{1}{M}} > 0$,

$$0 < |x+5| < \delta \Rightarrow 0 < |x+5|^2 < \delta^2 = \frac{1}{M} \Rightarrow \frac{1}{(x+5)^2} > M$$

93. (a) $\lim_{x \rightarrow c^-} f(x) = \infty \Leftrightarrow$ For any $M > 0$, there exists a $\delta > 0$ s.t.
if $-\delta < x-c < 0$ then $f(x) > M$

(b) $\lim_{x \rightarrow c^+} f(x) = -\infty \Leftrightarrow$ For any $-M < 0$, there exists a $\delta > 0$ s.t.
if $0 < x-c < \delta$ then $f(x) < -M$

(c) $\lim_{x \rightarrow c^-} f(x) = -\infty \Leftrightarrow$ For any $-M < 0$, there exists a $\delta > 0$ s.t.
if $-\delta < x-c < 0$ then $f(x) < -M$