

2. If $\lim_{x \rightarrow c} f(x) = L$ and $\lim_{x \rightarrow c} g(x) = M$

For any $\epsilon > 0$, there exists $\delta_1 > 0$ and $\delta_2 > 0$, such that

$$0 < |x - c| < \delta_1 \Rightarrow |f(x) - L| < \frac{\epsilon}{6}$$

$$0 < |x - c| < \delta_2 \Rightarrow |g(x) - M| < \frac{\epsilon}{6}$$

$$\Leftrightarrow \begin{cases} -\frac{2}{3}\epsilon < 4f(x) - 4L < \frac{2}{3}\epsilon \\ -\frac{1}{3}\epsilon < -2g(x) + 2M < \frac{1}{3}\epsilon \end{cases}$$

take $\delta = \min(\delta_1, \delta_2)$

$$0 < |x - c| < \delta \Rightarrow -\epsilon < 4f(x) - 2g(x) - (4L - 2M) < \epsilon$$

$$\Leftrightarrow |4f(x) - 2g(x) - (4L - 2M)| < \epsilon$$

$$\Rightarrow \lim_{x \rightarrow c} (4f(x) - 2g(x)) = 4L - 2M$$