$\boxed{\text{HW W3 =1}}$ If $\lim_{x\to c} f(x) = L$, $\lim_{x\to c} g(x) = M$.

For any $\varepsilon > 0$, there exists $\delta_1 > 0$ and $\delta_2 > 0$, such that

$0 < |x-c| < \delta_1 \Rightarrow -\varepsilon/2 < f(x) - L < \frac{\varepsilon}{2}$

$0 < |x-c| < \delta_2 \Rightarrow -\varepsilon/2 < g(x) - M < \frac{\varepsilon}{2}$

$\Rightarrow$ take $\delta = \min(\delta_1, \delta_2)$

$$\Rightarrow \begin{cases} -2\varepsilon < 4f(x) - 4L < 2\varepsilon \\ -\varepsilon < 2g(x) - 2M < \varepsilon \end{cases} \cancel{\Rightarrow} -\varepsilon < 4f(x) - 2g(x) - 4L + 2M < \varepsilon$$

$\Rightarrow -\varepsilon < 4f(x) - 2g(x) - (4L - 2M) < \varepsilon$

$\Rightarrow |4f(x) - 2g(x) - (4L-2M)| < \varepsilon \Rightarrow \lim_{m\to c}(4f(x) - 2g(x)) = 4L - 2M$

$\varepsilon > -2g(x) + 2M > -\varepsilon$

$\Rightarrow -2\varepsilon - \varepsilon < 4f(x) - 2g(x) - (4L - 2M) < 2\varepsilon + \varepsilon$