

## Brief solutions to selected problems in homework week 15

### 1. Section 9.2, problems 4:

9.2 4

$$P(x) = \tan x$$

$$Q(x) = \cos^3 x$$

$$v(x) = e^{\int \tan x dx} = e^{-\ln|\cos x|} = |\cos x|^{-1} = \sec x$$

$$y' + \frac{1}{\sec x} \sec x \cos^3 x dx = \cos x \int \frac{1}{\cos x} \cos^3 x dx$$

$$= \cos x \int \cos^2 x dx = \cos x (\sin x + C)$$

$$= \sin x \cos x + C \cos x \quad \square$$

$\Rightarrow (\sec x y)' = \cos x$

Figure 1: Section 9.2, problems 4.

**Remark:** One may assume that  $\ln|\cos x| = \ln \cos x$ . This amounts to finding the multiplication factor on the region  $\cos x > 0$ . Since  $|\cos x| = \pm \cos x$ , the effect of multiplying every term by  $\frac{1}{\cos x}$  is the same as multiplying every term by  $\frac{-1}{\cos x}$  and makes no difference in subsequent computations.

### 2. Section 7.3, problems 11, 37:

7.3 11  $\sinh(x+y)$  11  $\cosh$

$$\frac{e^{x+y} - e^{-(x+y)}}{2} = \frac{e^x e^y - e^{-x} e^{-y}}{4} \times 2$$

$$= \frac{(e^x e^y - e^{-x} e^{-y}) + (e^x e^{-y} - e^{-x} e^y)}{4}$$

$$= \sinh x \cosh y + \cosh x \sinh y$$

Figure 2: Section 7.3, problems 11, part 1.

7.3

11  $\cosh(x+y) = \cosh x \cdot \cosh y + \sinh x \cdot \sinh y$

$$\frac{e^{x+y} + e^{-(x+y)}}{2} = \left( \frac{e^x + e^{-x}}{2} \right) \left( \frac{e^y + e^{-y}}{2} \right) + \left( \frac{e^x - e^{-x}}{2} \right) \left( \frac{e^y - e^{-y}}{2} \right)$$

$$= \frac{e^{x+y} + e^{x-y} + e^{-x+y} + e^{-x-y}}{4} + \frac{e^{x+y} - e^{x-y} - e^{-x+y} + e^{-x-y}}{4}$$

$$= \frac{(e^{x+y} + e^{-(x+y)})}{2} = \cosh(x+y)$$

Figure 3: Section 7.3, problems 11, part 2.

7.3 11 (a) (b)

$\sinh(2x)$

$$a. = \sinh(x+x) = \sinh x \cosh x + \cosh x \sinh x$$

$$= 2 \sinh x \cosh x$$

$\cosh(2x)$

$$b. = \cosh(x+x) = \cosh x \cosh x + \sinh x \sinh x$$

$$= \cosh^2 x + \sinh^2 x$$

Figure 4: Section 7.3, problems 11, part 3.

7.3 37 (a)

$$\int \operatorname{sech} x \, dx = \tan^{-1}(\sinh x) + c$$

$$\ast \int \frac{1}{\cosh x} \, dx = \tan^{-1}(\sinh x) + c$$

$$\ast \frac{1}{\cosh x} = \frac{d}{dx} \tan^{-1}(\sinh x) + \frac{d}{dx} c$$

$$= \frac{1}{1 + \sinh^2 x} \cdot \cosh x = \frac{1}{\cosh x}$$

$\cosh^2 x - \sinh^2 x = 1$

Figure 5: Section 7.3, problems 37(a).

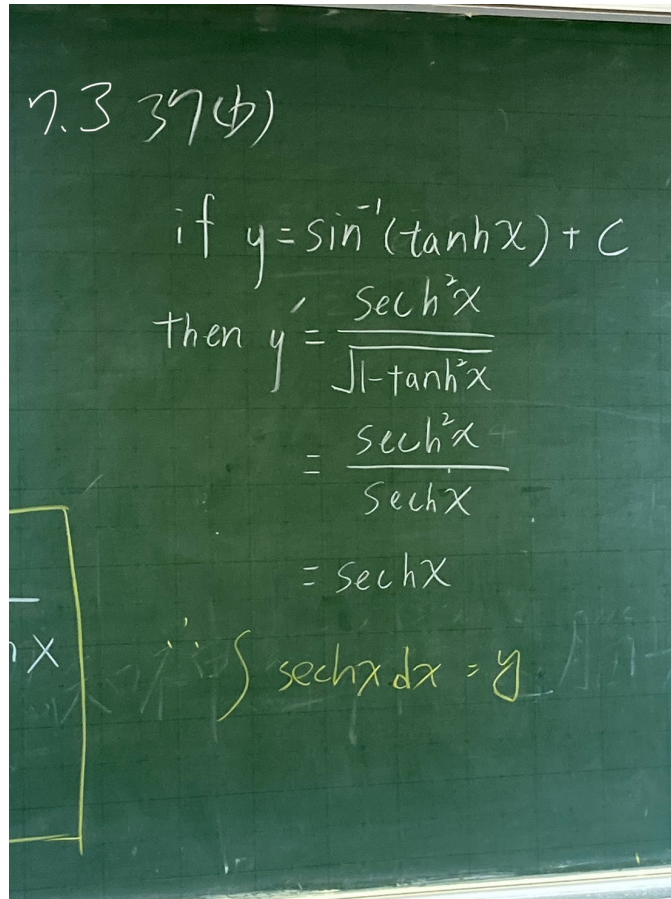


Figure 6: Section 7.3, problems 37(b).

3. Section 7.4, problems 10, 24:

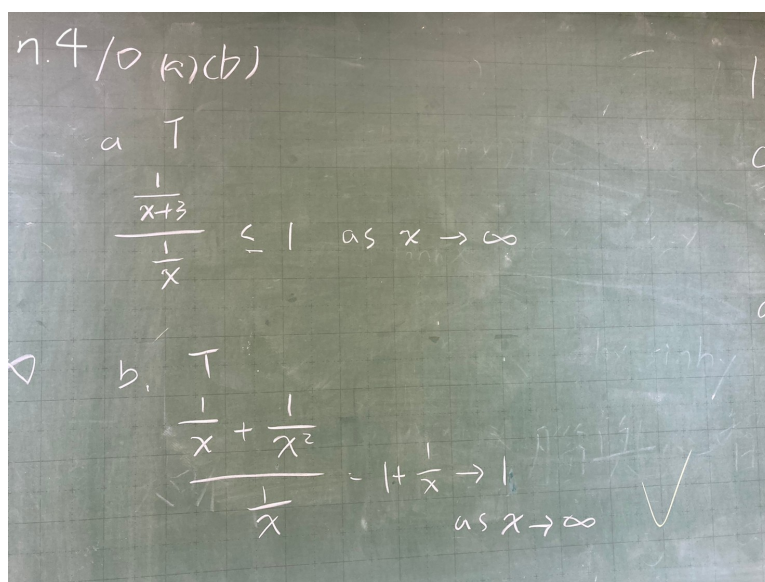


Figure 7: Section 7.4, problems 10 (a,b).

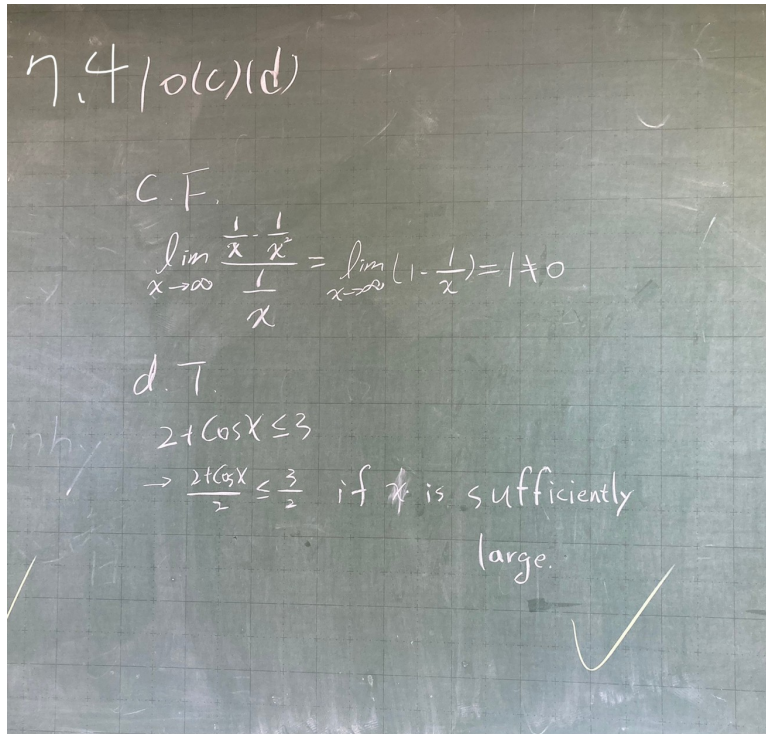


Figure 8: Section 7.4, problems 10 (c,d).

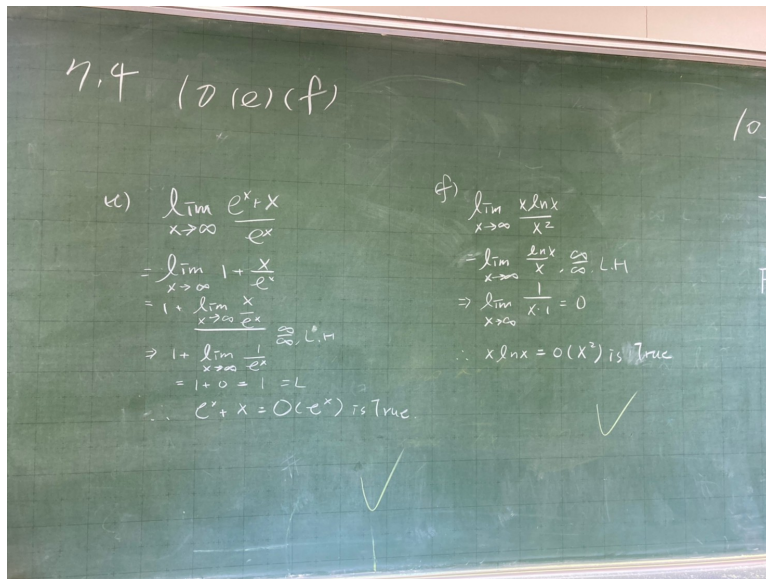


Figure 9: Section 7.4, problems 10 (e,f).

7.4/10 (a)(b)

a.  $T$

$$\frac{\frac{1}{x+3}}{\frac{1}{x}} \leq 1 \text{ as } x \rightarrow \infty$$

b.  $T$

$$\frac{\frac{1}{x} + \frac{1}{x^2}}{\frac{1}{x}} \rightarrow 1 + \frac{1}{x} \rightarrow 1 \text{ as } x \rightarrow \infty$$

Figure 10: Section 7.4, problems 10 (g,h).

7.4  
24

$n, \sqrt{n} \log_2 n, (\log_2 n)^2$

$$\lim_{n \rightarrow \infty} \frac{n}{\sqrt{n} \log_2 n} = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\log_2 n} = \infty$$

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\log_2 n} \stackrel{\text{L'H}}{=} \lim_{n \rightarrow \infty} \frac{1/2 \cdot \sqrt{n}}{\log_2 e \cdot 1/n} = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{2 \log_2 e} = \infty$$

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n} \log_2 n}{(\log_2 n)^2} = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\log_2 n} = \infty$$

rate growth  
 $n > \sqrt{n} \log_2 n > (\log_2 n)^2$   
 $\Rightarrow (\log_2 n)^2$

$\log_2 n = \log_2 e \cdot \ln n$

Figure 11: Section 7.4, problems 24.