

### Brief solutions to selected problems in homework week 14

1. Section 6.3, problems 25, 29. Section 6.4, problem 23.

**Sec 6.3**

25.  $y(x) = \int_0^x \sqrt{\cos 2t} dt$  by Fundamental thm of Calculus  
 $\Rightarrow y'(x) = \sqrt{\cos 2x}$   
 $\Rightarrow \int_0^{\frac{\pi}{4}} \sqrt{1+(y')^2} dx = \int_0^{\frac{\pi}{4}} \sqrt{1+\cos 2x} dx = \int_0^{\frac{\pi}{4}} \sqrt{2\cos^2 x} dx$   
 $(\because \cos x \geq 0 \text{ if } x \in [0, \frac{\pi}{4}]) = \sqrt{2} \int_0^{\frac{\pi}{4}} \cos x dx = \sqrt{2} \sin x \Big|_0^{\frac{\pi}{4}} = 1$

29.  $9x^2 = y(y-3)^2 \Rightarrow 18x dx = [(y-3)^2 + 2y(y-3)] dy$   
 代入  $\Rightarrow 6x dx = (y-3)(y-1) dy$   
 $\Rightarrow 36x^2 dx^2 = (y-3)^2 (y-1)^2 dy^2$   
 $\Rightarrow dx^2 = \frac{(y-3)^2 (y-1)^2}{4 \cdot y (y-3)^2} dy^2$   
 $ds^2 = dx^2 + dy^2$   
 $= \frac{(y-1)^2}{4y} dy^2 + dy^2 = \left[ \frac{(y-1)^2 + 4y}{4y} \right] dy^2 = \frac{(y+1)^2}{4y} dy^2$

**Sec 6.4**

23.  $\int_{y=1}^{y=2} 2\pi y ds = \int_{y=1}^{y=2} 2\pi y \sqrt{dx^2 + dy^2} = \int_{y=1}^{y=2} 2\pi y \sqrt{\left(\frac{dx}{dy}\right)^2 + 1} dy$   
 $\Rightarrow S = \int_{y=1}^{y=2} 2\pi y \sqrt{(x')^2 + 1} dy$   
 $= \int_{y=1}^{y=2} 2\pi y \sqrt{\left(y^3 - \frac{1}{4y^3}\right)^2 + 1} dy \quad (x' = y^3 - \frac{1}{4y^3})$   
 $= \int_1^2 2\pi y \sqrt{\left(y^3 + \frac{1}{4y^3}\right)^2} dy$   
 $= \int_1^2 2\pi y \left[y^3 + \frac{1}{4y^3}\right] dy \quad (\because y^3 + \frac{1}{4y^3} \geq 0 \text{ if } y \in [1, 2])$   
 $= 2\pi \left( \frac{y^5}{5} - \frac{1}{4y} \Big|_1^2 \right) = \frac{253}{20} \pi$

Figure 1: Section 6.3, problems 25, 29. Section 6.4, problem 23