

Brief solutions to selected problems in homework week 10

1. Section 4.4:

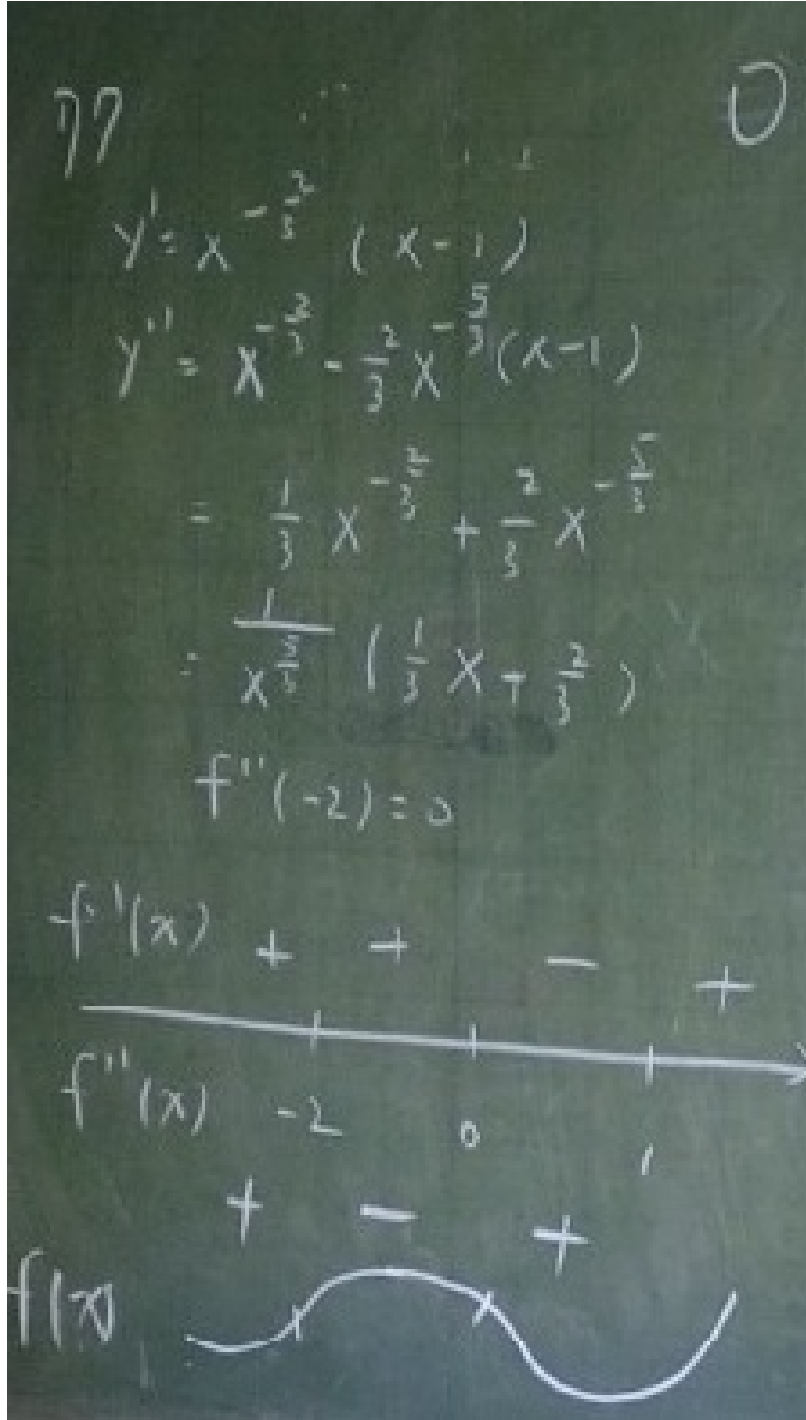


Figure 1: Section 4.4, problem 77

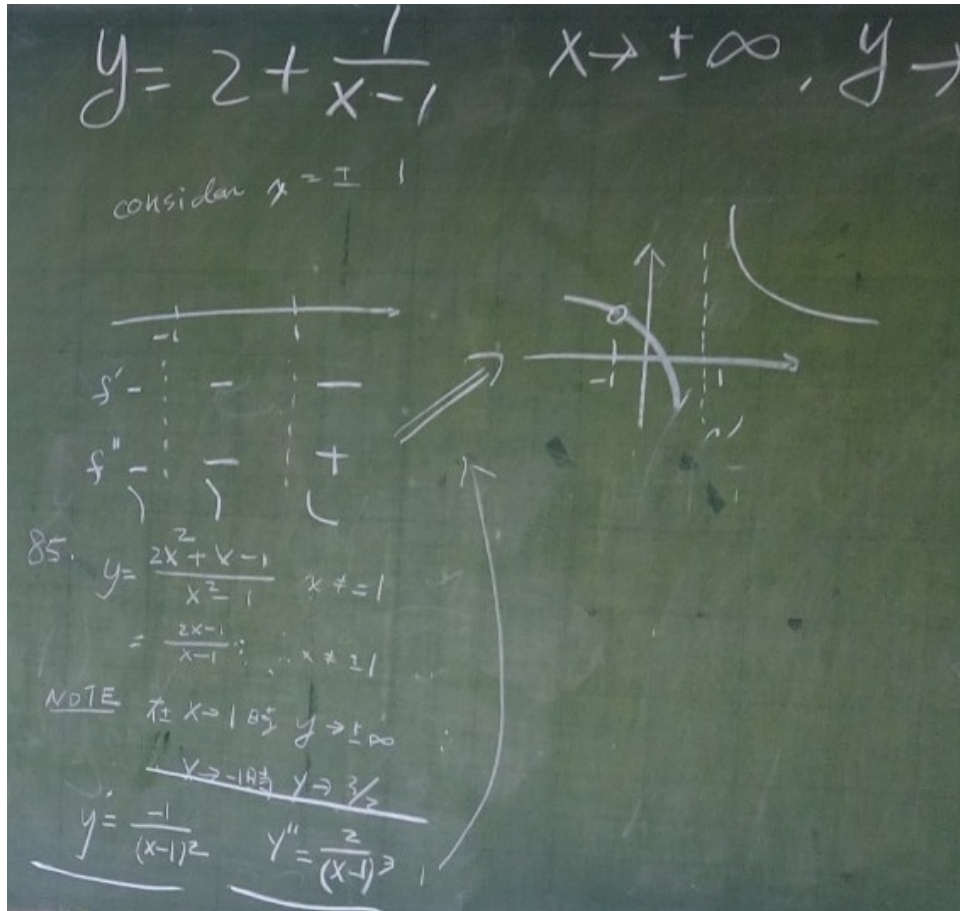


Figure 2: Section 4.4, problem 85

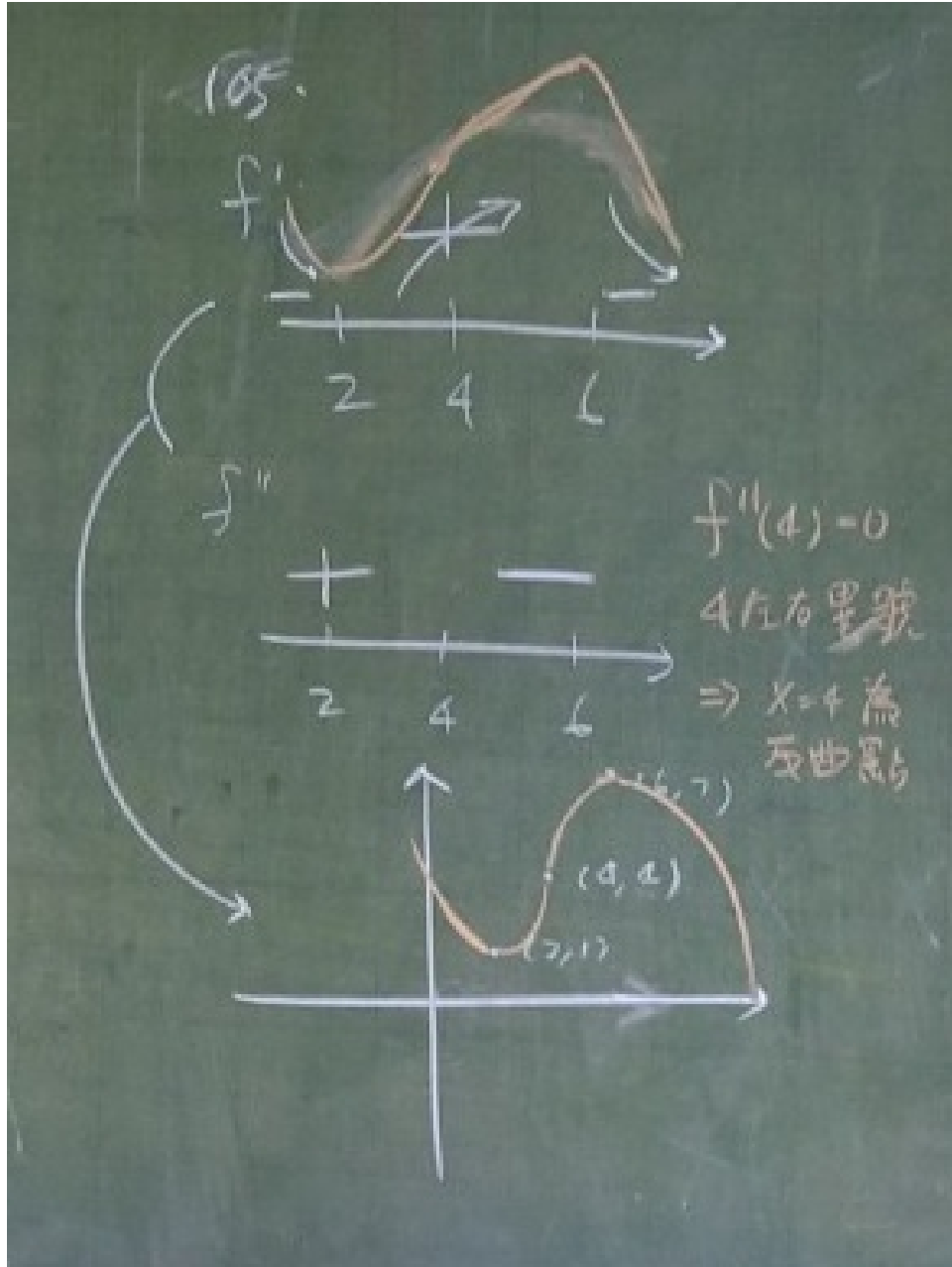


Figure 3: Section 4.4, problem 105

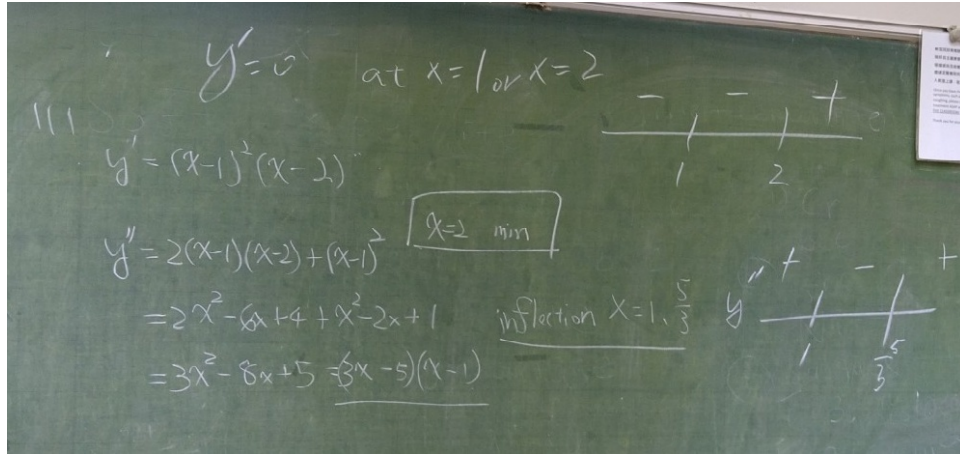


Figure 4: Section 4.4, problem 111

2. Section 4.5:

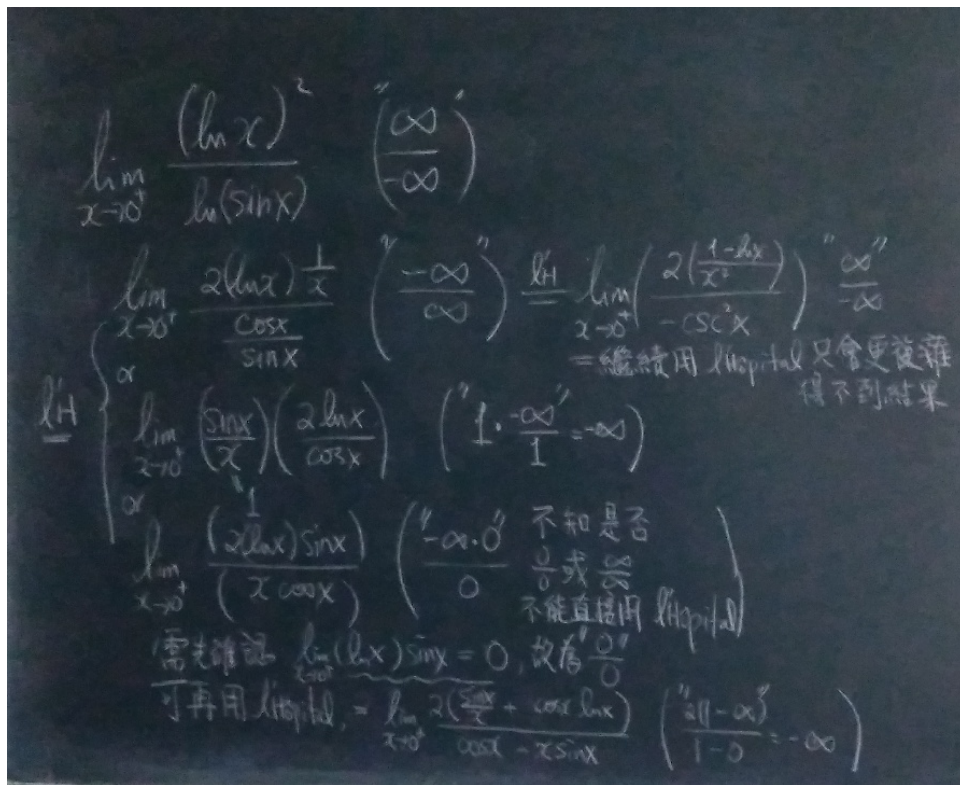


Figure 5: Section 4.5, problem 39

73.

$$\lim_{x \rightarrow \infty} \frac{e^{x+2}}{xe^x} = \lim_{x \rightarrow \infty} \frac{e^{x^2-x}}{x}$$

$$= \lim_{x \rightarrow \infty} \frac{e^{x(x-1)}}{x}$$

$$= \lim_{x \rightarrow \infty} \frac{e^{x(x-1)} \cdot (2x-1)}{1} = \infty$$

Figure 6: Section 4.5, problem 73

$$\lim_{x \rightarrow 0} \frac{\tan 2x + ax}{x^3} = -6$$

$$\lim_{x \rightarrow 0} \frac{2 \sec^2 2x + a}{3x^2} \quad \left(\lim_{x \rightarrow 0} \sec^2 2x = a = -2 \right)$$

$$\lim_{x \rightarrow 0} \frac{2}{3} \frac{\sin^2 2x}{x^2} \frac{1}{\cos^2 2x} = \frac{8}{3}$$

Figure 7: Section 4.5, problem 80

$$f(x) = e^{\ln f(x)} = e^{\frac{x \ln(1 + \frac{1}{x^2})}{1}}$$

84(c)

Let $f(x) = (1 + \frac{1}{x^2})^x \Rightarrow \ln f(x) = \frac{1}{x} \ln(1 + \frac{1}{x^2})$

$$\Rightarrow \lim_{x \rightarrow \infty} x \ln(1 + \frac{1}{x^2}) = \lim_{x \rightarrow \infty} \frac{\ln(1 + \frac{1}{x^2})}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\frac{1}{1 + \frac{1}{x^2}} \cdot \left(\frac{-2}{x^3}\right)}{-\frac{1}{x^2}}$$

$$= \lim_{x \rightarrow \infty} \frac{2}{x} \left(\frac{1}{1 + \frac{1}{x^2}} \right) = 0$$

Figure 8: Section 4.5, problem 84(c)

$$f'(0) = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x \rightarrow 0} \frac{e^{-\frac{1}{x^2}}}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{x}}{e^{\frac{1}{x^2}}} \xrightarrow{\text{L'H}} \lim_{x \rightarrow 0} \frac{-\frac{1}{x^2}}{\frac{2}{x^3}} = \lim_{x \rightarrow 0} \frac{x}{2e^{\frac{1}{x^2}}}$$

By ~~~~~

Figure 9: Section 4.5, problem 88