

Brief solutions to selected problems in homework week 10

1. Section 4.4:

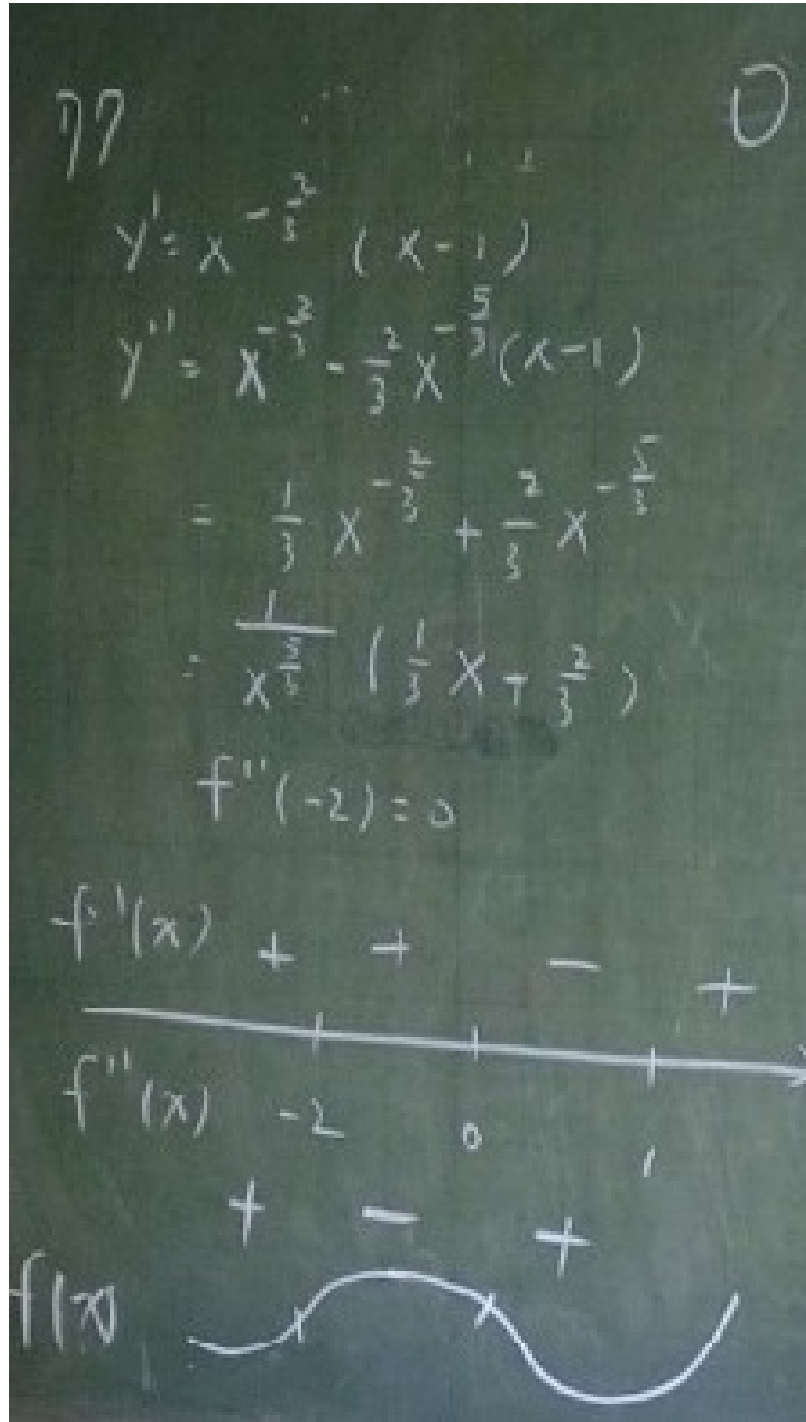


Figure 1: Section 4.4, problem 77

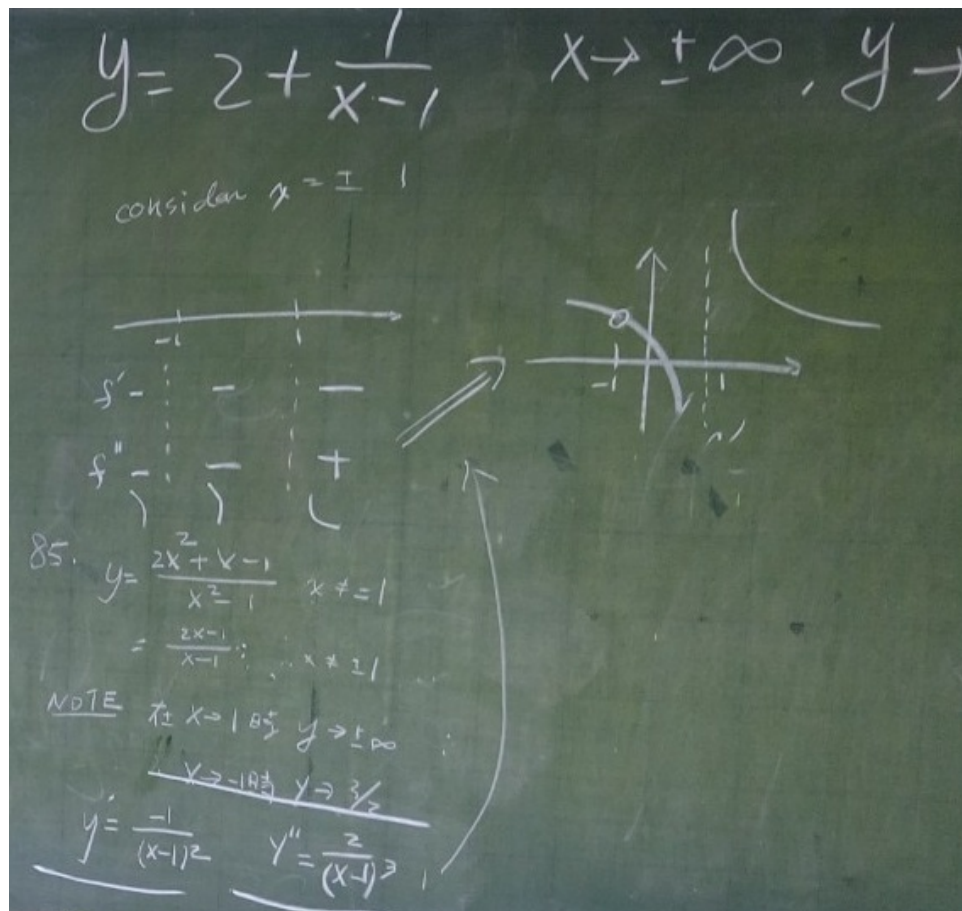


Figure 2: Section 4.4, problem 85

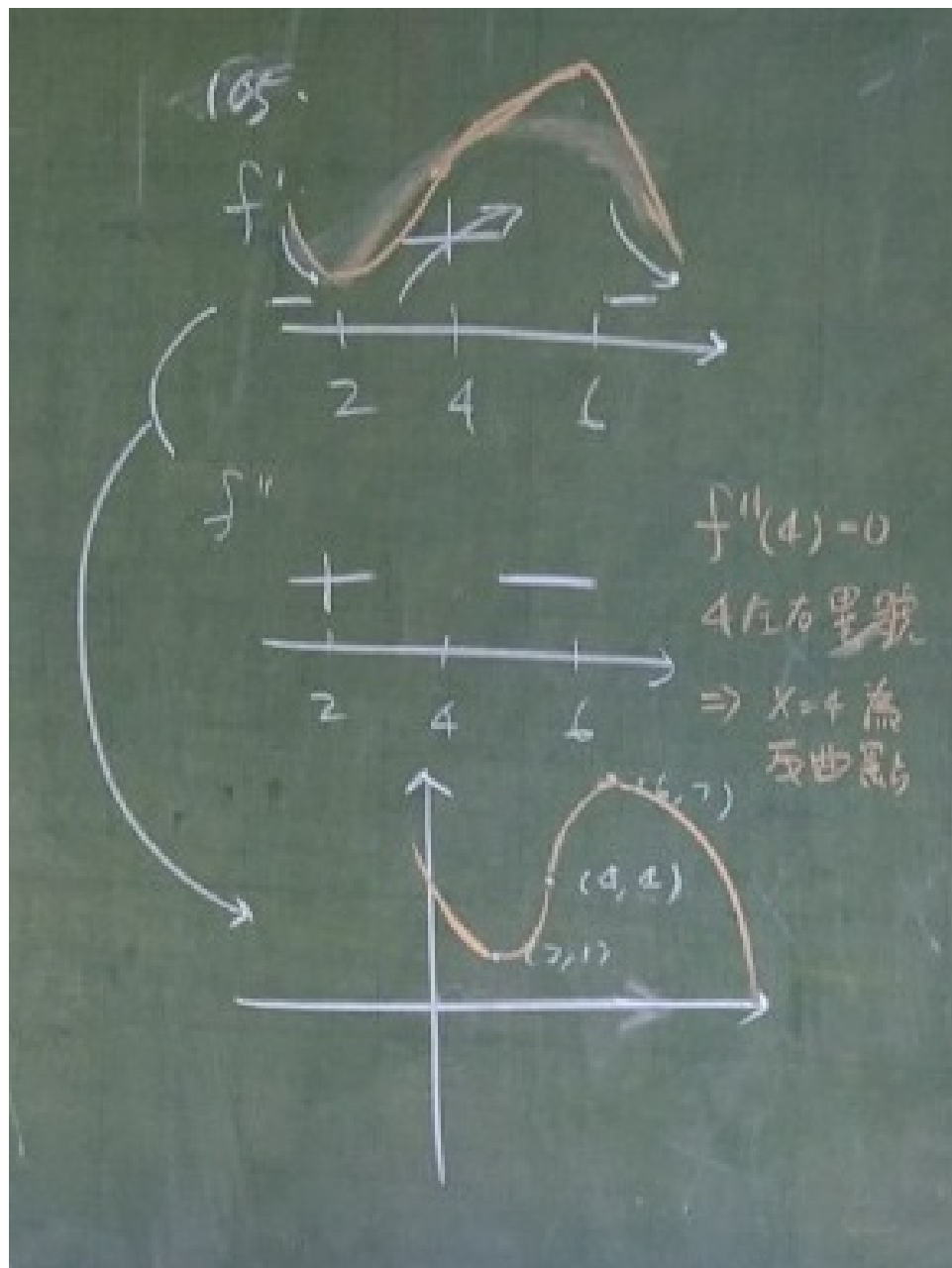


Figure 3: Section 4.4, problem 105

$y' = 0$ at $x=1$ or $x=2$

111 $y' = (x-1)^2(x-2)$

$y'' = 2(x-1)(x-2) + (x-1)^2$ $x=2$ min

$= 2x^2 - 6x + 4 + x^2 - 2x + 1$ inflection $x=1, \frac{5}{3}$

$= 3x^2 - 8x + 5 = (x-5)(x-1)$

Figure 4: Section 4.4, problem 111

2. Section 4.5:

73.

$$\begin{aligned}
 \lim_{x \rightarrow \infty} \frac{e^{x^2}}{xe^x} &= \lim_{x \rightarrow \infty} \frac{e^{x^2-x}}{x} \\
 &= \lim_{x \rightarrow \infty} \frac{e^{x(x-1)}}{x} \\
 &= \lim_{x \rightarrow \infty} \frac{e^{x(x-1)} \cdot (2x-1)}{1} = \infty
 \end{aligned}$$

Figure 5: Section 4.5, problem 73

$$\lim_{x \rightarrow 0} \frac{\tan 2x + ax}{x^3} = -b$$

$$\lim_{x \rightarrow 0} \frac{2 \sec^2 2x + a}{3x^2} \quad \left(\lim_{x \rightarrow 0} \sec^2 2x = \infty \Rightarrow a = -2 \right)$$

$$\lim_{x \rightarrow 0} \frac{2}{3} \frac{\sin^2 2x}{x^2} \frac{1}{\cos^2 2x} = \frac{8}{3}$$

Figure 6: Section 4.5, problem 80

$$f(x) = e^{\ln f(x)} = e^{\frac{x \ln(1 + \frac{1}{x^2})}{1}}$$

84(c)
 Let $f(x) = (1 + \frac{1}{x^2})^x \Rightarrow \ln f(x) = \frac{1}{x} \ln(1 + \frac{1}{x^2})$

$$\Rightarrow \lim_{x \rightarrow \infty} x \ln(1 + \frac{1}{x^2}) = \lim_{x \rightarrow \infty} \frac{\ln(1 + \frac{1}{x^2})}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\frac{1}{1 + \frac{1}{x^2}} \cdot \left(\frac{-2}{x^3} \right)}{-\frac{1}{x^2}} = \lim_{x \rightarrow \infty} \frac{2}{x} \left(\frac{1}{1 + \frac{1}{x^2}} \right) = 0$$

Figure 7: Section 4.5, problem 84(c)

$$\begin{aligned}
 f'(0) &= \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0} \\
 &= \lim_{x \rightarrow 0} \frac{e^{-\frac{1}{x^2}}}{x} \\
 &= \lim_{x \rightarrow 0} \frac{\frac{1}{x}}{e^{\frac{1}{x^2}}} \rightarrow \lim_{x \rightarrow 0} \frac{-\frac{1}{x^2}}{e^{\frac{1}{x^2}} \left| -\frac{2}{x^3} \right|} = \lim_{x \rightarrow 0} \frac{x}{e^{\frac{1}{x^2} \cdot 2}}
 \end{aligned}$$

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Figure 8: Section 4.5, problem 88