

Brief solutions to selected problems in homework week 08

1. Section 3.9, problem 53:

Note: To conclude with the $|x|$ factor, one needs to go into details about the domains and ranges of \sec and \sec^{-1} . This step was overlooked by most. See page 5-6 of Lecture 11 note on how to do this properly for \csc^{-1} .

2. Section 3.11, problem 66:

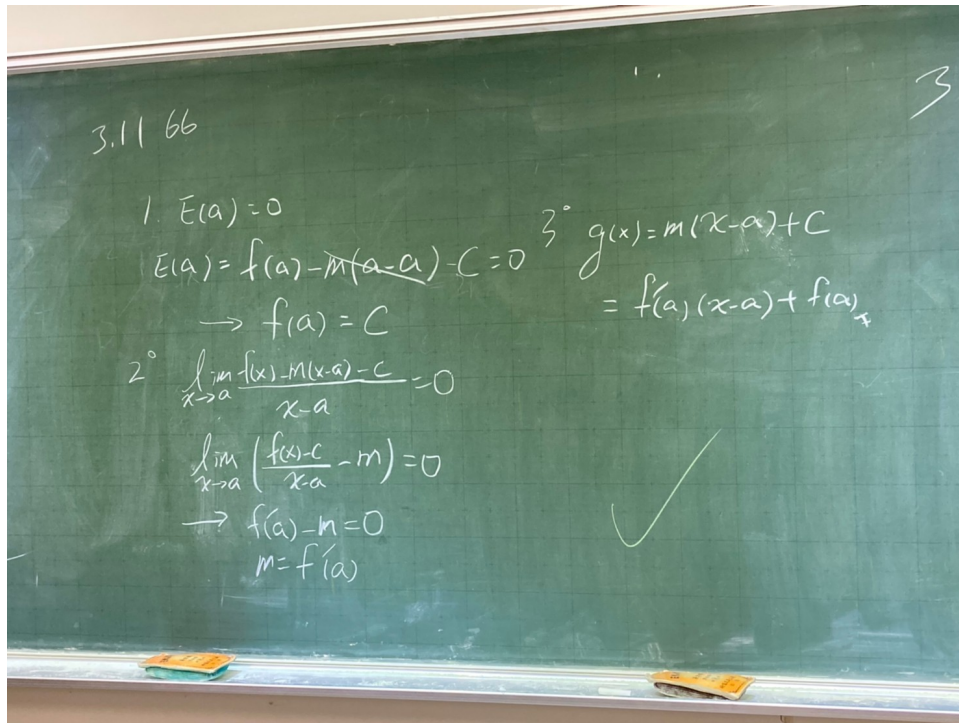


Figure 1: Section 3.11, problem 66

3. Problem 3:

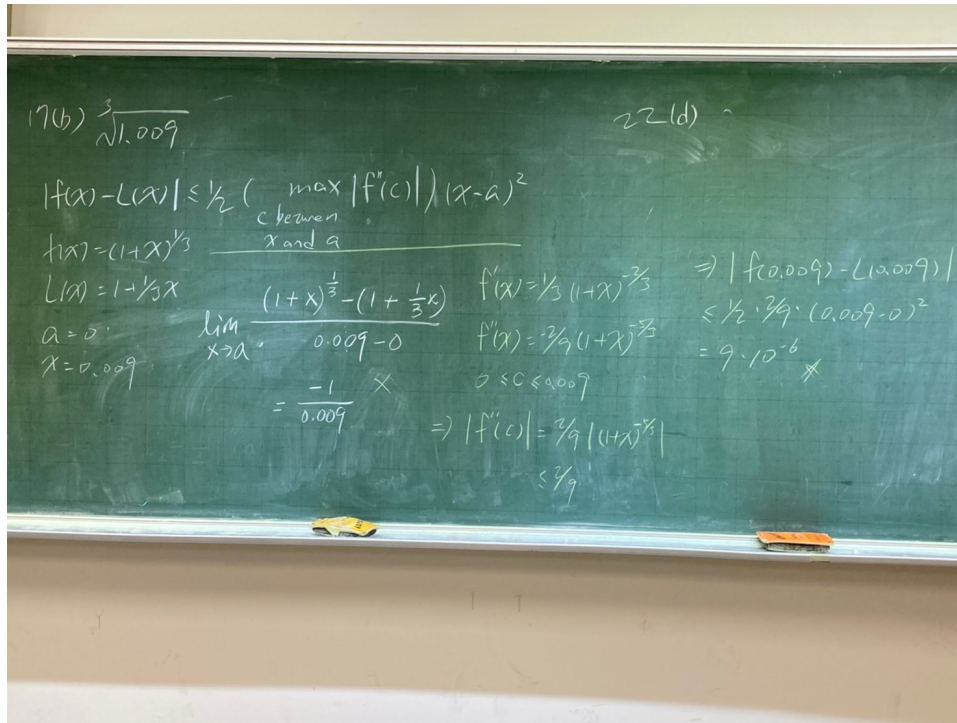


Figure 2: Problem 3

Remark: It is in fact not obvious that (2) implies (1), unless one makes additional assumptions, such as "f'' is continuous at a". Please ignore this comment (between (2) and (3)).

4. Chap 03, additional and advanced problem 16:

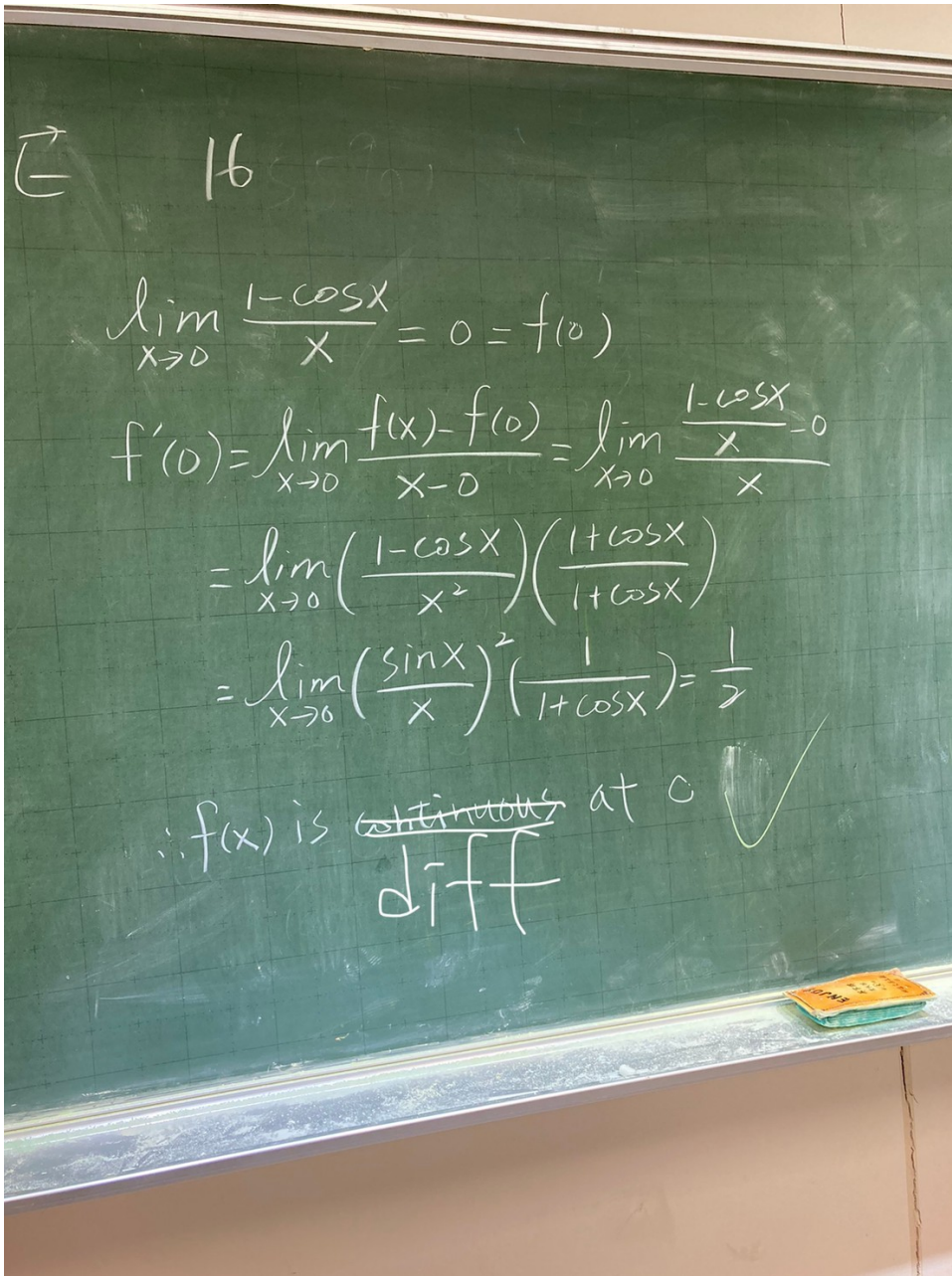


Figure 3: Chap 03, additional and advanced problem 16

5. Chap 03, additional and advanced problem 21:

$$\begin{aligned}
 & 21) f(x)g(x) - f(x_0)g(x_0) \\
 & = f(x)g(x) - f(x_0)g(x) + f(x_0)g(x) - f(x_0)g(x_0) \\
 & \lim_{x \rightarrow x_0} \frac{f(x)g(x) - f(x_0)g(x_0)}{x - x_0} \\
 & = \lim_{x \rightarrow x_0} \frac{g(x)[f(x) - f(x_0)]}{x - x_0} + \lim_{x \rightarrow x_0} \left[f(x_0) \cdot \frac{g(x) - g(x_0)}{x - x_0} \right] \\
 & = \lim_{x \rightarrow x_0} g(x) \cdot f'(x) + 0 = g(x_0) f'(x) \quad \because g(x) \text{ is diff in } f
 \end{aligned}$$

Figure 4: Chap 03, additional and advanced problem 21

6. Chap 03, additional and advanced problem 22(d):

$$\begin{aligned}
 & 22(d) \\
 & h(x) = \begin{cases} x^2 \sin 1/x & x \neq 0 \\ 0 & x = 0 \end{cases} \\
 & f(x) = x, \quad f \text{ diff at } x=0, \quad f'(0) = 1 \\
 & g(x) = \begin{cases} x \sin 1/x & x \neq 0 \\ 0 & x = 0 \end{cases} \quad g \text{ is contin at } 0 \\
 & h = fg \\
 & \therefore \text{ by 21, } h \text{ is diff at } x=0 \quad \#
 \end{aligned}$$

Figure 5: Chap 03, additional and advanced problem 22(d)