Brief solutions to selected problems in homework week 04

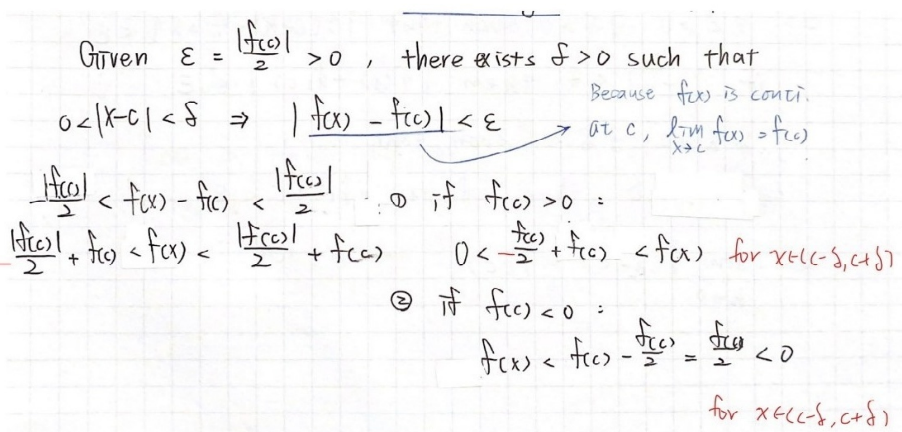
Given $\varepsilon = \frac{|f(c)|}{2} > 0$, there exists $\delta > 0$ such that
$0<|x-c|<\delta \Rightarrow |f(x)-f(c)|<\varepsilon$
Because f(x) is conti. at c, $\lim_{x\to c} f(x) = f(c)$
$-\frac{|f(c)|}{2} < f(x)-f(c) < \frac{|f(c)|}{2}$
$-\frac{|f(c)|}{2}+f(c) < f(x) < \frac{|f(c)|}{2}+f(c)$
① if $f(c)>0$:
$0 < -\frac{f(c)}{2}+f(c) < f(x)$ for $x\in(c-\delta,c+\delta)$
② if $f(c)<0$:
$f(x) < f(c) - \frac{f(c)}{2} = \frac{f(c)}{2} < 0$
for $x\in(c-\delta,c+\delta)$

Figure 1: Correct answer section 2.5, problem 68 (only minor mistake)

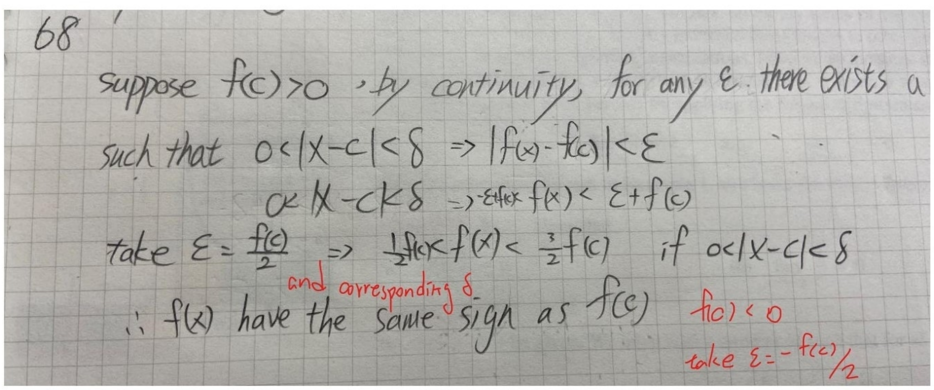
68
suppose $f(c)>0$, by continuity, for any ε there exists a
such that $0<|x-c|<\delta \Rightarrow |f(x)-f(c)|<\varepsilon$
$0<|x-c|<\delta \Rightarrow -\varepsilon+f(c) < f(x) < \varepsilon+f(c)$
take $\varepsilon = \frac{f(c)}{2} \Rightarrow \frac{1}{2}f(c) < f(x) < \frac{3}{2}f(c)$ if $0<|x-c|<\delta$
and corresponding δ
$\therefore$ f(x) have the same sign as f(c)
$f(c)<0$
take $\varepsilon = -f(c)/2$

Figure 2: A common mistake of section 2.5, problem 68. One should at least state how to choose ε when $f(c) < 0$