

Brief solutions to selected problems in homework week 02 and week 03

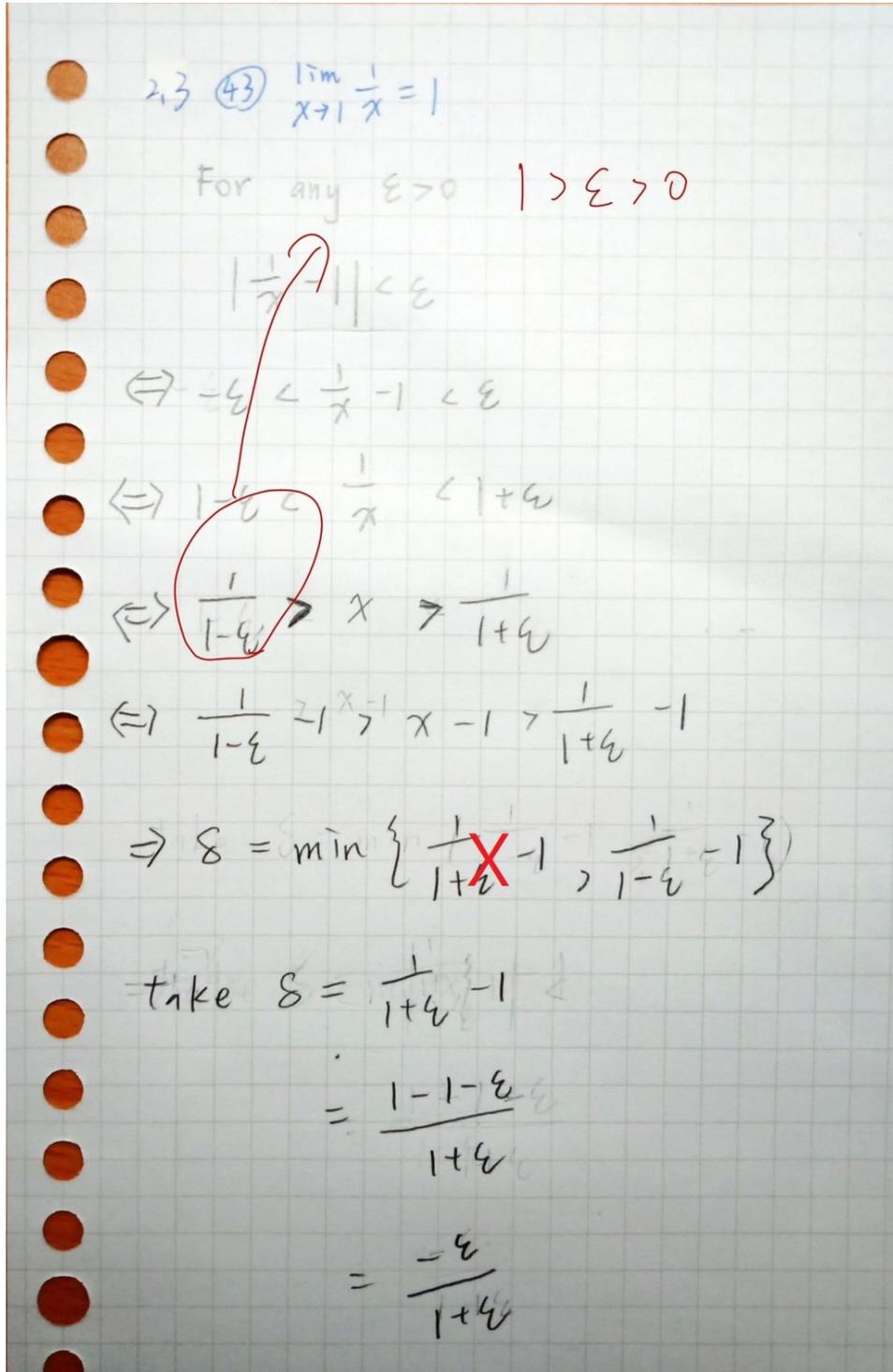


Figure 1: Section 2.3, problem 43: mistake 1

43.

$$0 < |x-1| < \delta$$

$$\Rightarrow -\delta < x-1 < \delta$$

$$\Rightarrow -\delta+1 < x < \delta+1$$

$$|\frac{1}{x}-1| < \epsilon \quad (A)$$

$$\Leftrightarrow -\epsilon < \frac{1}{x}-1 < \epsilon$$

$$\Leftrightarrow -\epsilon+1 < \frac{1}{x} < \epsilon+1 \quad \text{assume } 0 < \epsilon < 1$$

$$\Leftrightarrow \frac{1}{1+\epsilon} < x < \frac{1}{1-\epsilon} \quad (B)$$

$$\begin{cases} \delta+1 \leq \frac{1}{1-\epsilon} \\ -\delta+1 \geq \frac{1}{1+\epsilon} \end{cases} \Rightarrow \begin{cases} \delta \leq \frac{1}{1-\epsilon} - 1 \\ \delta \leq 1 - \frac{1}{1+\epsilon} \end{cases}$$

$$\text{take } \delta = \min \left\{ \frac{1}{1-\epsilon} - 1, 1 - \frac{1}{1+\epsilon} \right\}$$

因為要說明所選取的delta可以從(B)推到(A),反向箭頭是必要的

Figure 2: Section 2.3, problem 43: mistake 2

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43. $\lim_{x \rightarrow 1} \frac{1}{x} = 1$

for any $\epsilon > 0$, there exist $\delta > 0$

$0 < |x-1| < \delta \Leftrightarrow \left| \frac{1}{x} - 1 \right| < \epsilon$

$\Leftrightarrow -\epsilon < \frac{1}{x} - 1 < \epsilon$

$\Leftrightarrow 1 - \epsilon < \frac{1}{x} < 1 + \epsilon$

$\Leftrightarrow \frac{1}{1 + \epsilon} < x < \frac{1}{1 - \epsilon}$

$\Leftrightarrow \frac{1 - (1 + \epsilon)}{1 - \epsilon} < |x-1| < \frac{1 - (1 - \epsilon)}{1 - \epsilon}$

$\Leftrightarrow \frac{\epsilon}{1 - \epsilon} < |x-1| < \frac{\epsilon}{1 - \epsilon}$

take $\delta = \min \left(\frac{\epsilon}{1 - \epsilon}, \frac{\epsilon}{1 - \epsilon} \right)$

$\Leftrightarrow 0 < |x-1| < \delta \Leftrightarrow \left| \frac{1}{x} - 1 \right| < \epsilon$

Figure 3: Section 2.3, problem 43: mistake 3

43.

prove $\lim_{x \rightarrow 1} \frac{1}{x} = 1$

$0 < |x - c| < \delta, |f(x) - L| < \epsilon$

Given $\epsilon > 0$, there exists $\delta > 0$.

$0 < |x - c| < \delta, |f(x) - L| < \epsilon$

$|\frac{1}{x} - 1| < \epsilon$ (A)

$\Leftarrow -\epsilon < \frac{1}{x} - 1 < \epsilon$

$\Leftarrow -\epsilon + 1 < \frac{1}{x} < \epsilon + 1$

$\Leftarrow \frac{1}{\epsilon + 1} > x > \frac{1}{\epsilon + 1}$ (B)

$-\delta < x - 1 < \delta$

$-\delta + 1 < x < \delta + 1$

$\begin{cases} 1 - \delta \geq \frac{1}{\epsilon + 1} \\ \delta + 1 \leq \frac{1}{1 - \epsilon} \end{cases}$

$\begin{cases} \delta \leq 1 - \frac{1}{1 + \epsilon} = \frac{\epsilon}{1 + \epsilon} \\ \delta \leq \frac{1}{1 - \epsilon} - 1 = \frac{\epsilon}{1 - \epsilon} \end{cases}$

$\therefore \delta = \min\left\{\frac{\epsilon}{1 + \epsilon}, \frac{\epsilon}{1 - \epsilon}\right\}$

$= \frac{\epsilon}{1 + \epsilon}$

因為要說明所選取的delta可以從(B)推到(A), 反向的箭頭是必須的

Figure 4: Section 2.3, problem 43: mistake 4

2. If $\lim_{x \rightarrow c} f(x) = L$ and $\lim_{x \rightarrow c} g(x) = M$

For any $\varepsilon > 0$, there exists $\delta_1 > 0$ and $\delta_2 > 0$, such that

$$0 < |x - c| < \delta_1 \Rightarrow |f(x) - L| < \frac{\varepsilon}{6}$$

$$0 < |x - c| < \delta_2 \Rightarrow |g(x) - M| < \frac{\varepsilon}{6}$$

$$\Leftrightarrow \begin{cases} -\frac{2}{3}\varepsilon < 4f(x) - 4L < \frac{2}{3}\varepsilon \\ -\frac{1}{3}\varepsilon < -2g(x) + 2M < \frac{1}{3}\varepsilon \end{cases}$$

take $\delta = \min(\delta_1, \delta_2)$

$$0 < |x - c| < \delta \Rightarrow -\varepsilon < 4f(x) - 2g(x) - (4L - 2M) < \varepsilon$$

$$\Leftrightarrow |4f(x) - 2g(x) - (4L - 2M)| < \varepsilon$$

$$\Rightarrow \lim_{x \rightarrow c} (4f(x) - 2g(x)) = 4L - 2M$$

Figure 5: Answer to homework week 03, problem 2