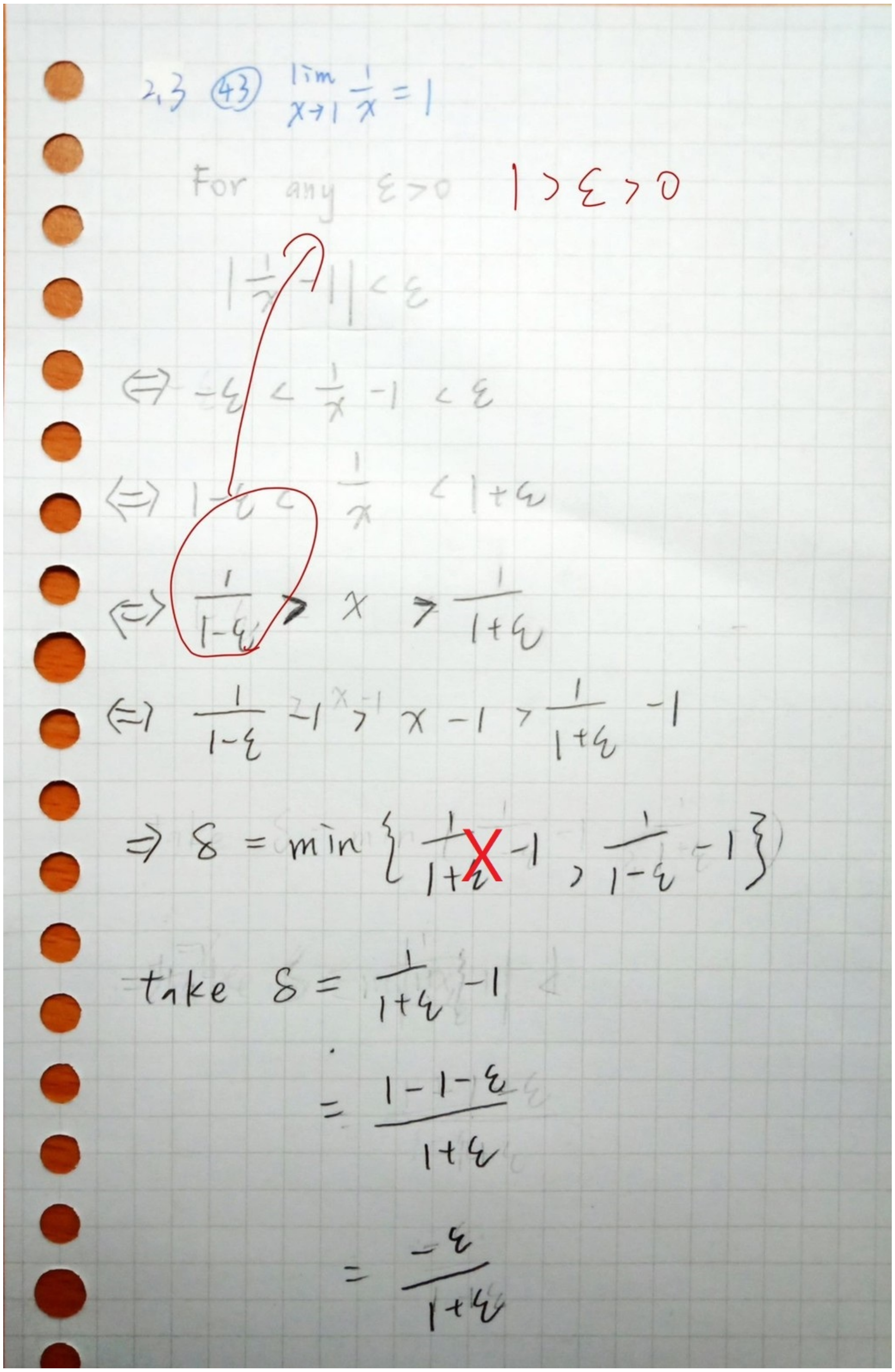
Brief solutions to selected problems in homework week 02 and week 03


2.3 (43) $\lim_{x\to 1} \frac{1}{x} = 1$

For any $\varepsilon > 0$ $\quad 1 > \varepsilon > 0$

$$\left|\frac{1}{x} - 1\right| < \varepsilon$$

$$\Leftrightarrow -\varepsilon < \frac{1}{x} - 1 < \varepsilon$$

$$\Leftrightarrow 1-\varepsilon < \frac{1}{x} < 1+\varepsilon$$

$$\Leftrightarrow \frac{1}{1-\varepsilon} > x > \frac{1}{1+\varepsilon}$$

$$\Leftrightarrow \frac{1}{1-\varepsilon} - 1 > x - 1 > \frac{1}{1+\varepsilon} - 1$$

$$\Rightarrow \delta = \min\left\{ \frac{1}{1+\varepsilon} - 1, \frac{1}{1-\varepsilon} - 1 \right\}$$

take $\delta = \frac{1}{1+\varepsilon} - 1$

$$= \frac{1-1-\varepsilon}{1+\varepsilon}$$

$$= \frac{-\varepsilon}{1+\varepsilon}$$

Figure 1: Section 2.3, problem 43: mistake 1

43.

$$0<|x-1|<\delta$$

$$\Rightarrow -\delta<x-1<\delta$$

$$\Rightarrow -\delta+1<x<\delta+1$$

$$\left|\frac{1}{x}-1\right|<\varepsilon \qquad \text{(A)}$$

$$\Leftrightarrow -\varepsilon<\frac{1}{x}-1<\varepsilon$$

$$\Leftrightarrow -\varepsilon+1<\frac{1}{x}<\varepsilon+1 \qquad \text{assume } 0<\varepsilon<1$$

$$\Leftrightarrow \frac{1}{1+\varepsilon}<x<\frac{1}{1-\varepsilon} \qquad \text{(B)}$$

$$\begin{cases} \delta+1 \le \frac{1}{1-\varepsilon} \\ -\delta+1 \ge \frac{1}{1+\varepsilon} \end{cases} \Rightarrow \begin{cases} \delta \le \frac{1}{1-\varepsilon}-1 \\ \delta \le 1-\frac{1}{1+\varepsilon} \end{cases}$$

$$\text{take } \delta=\min\left\{\frac{1}{1-\varepsilon}-1,\ 1-\frac{1}{1+\varepsilon}\right\}$$

因為要說明所選取的delta可以從(B)推到(A),反向箭頭是必要的

Figure 2: Section 2.3, problem 43: mistake 2

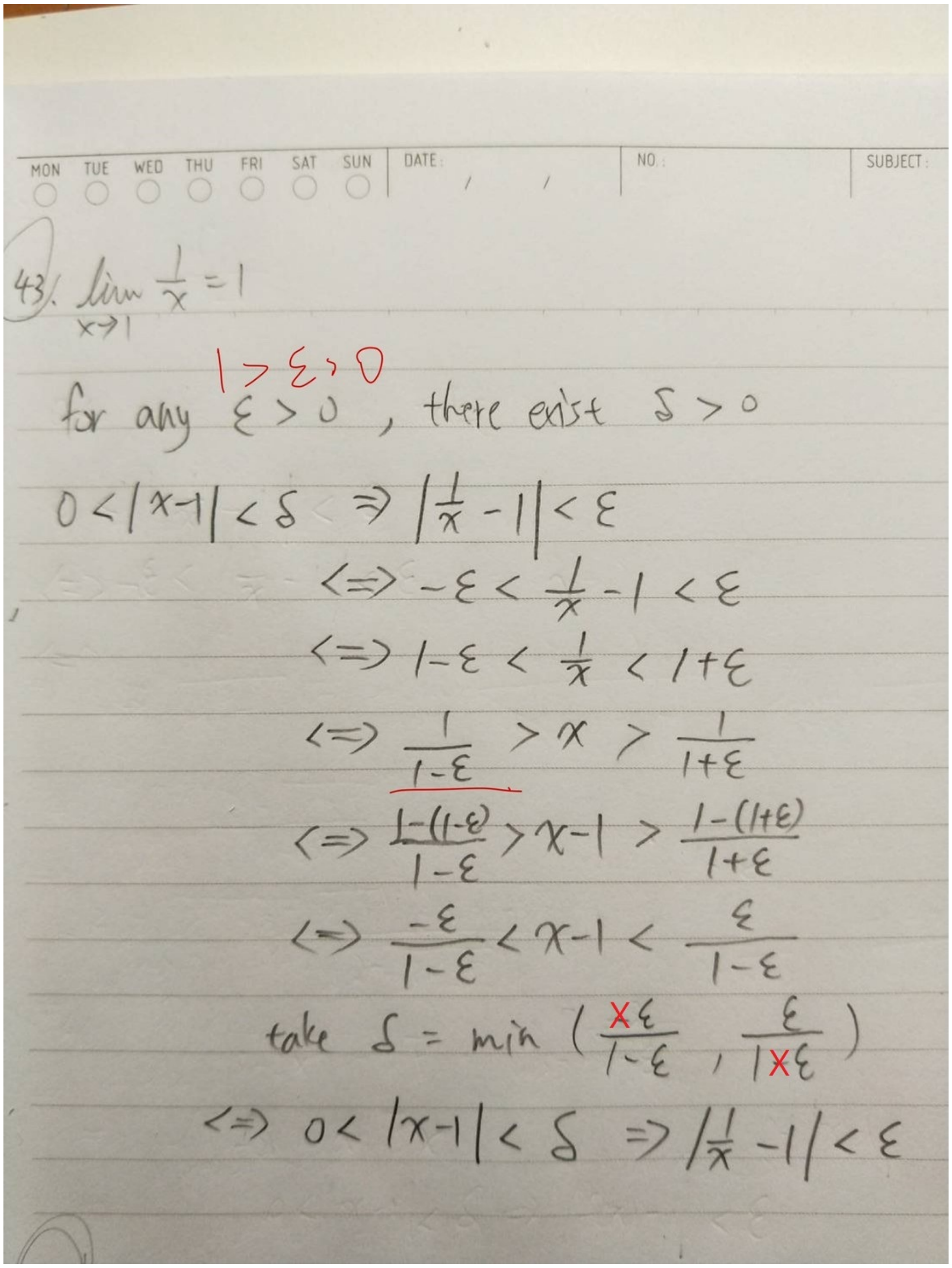


Figure 3: Section 2.3, problem 43: mistake 3

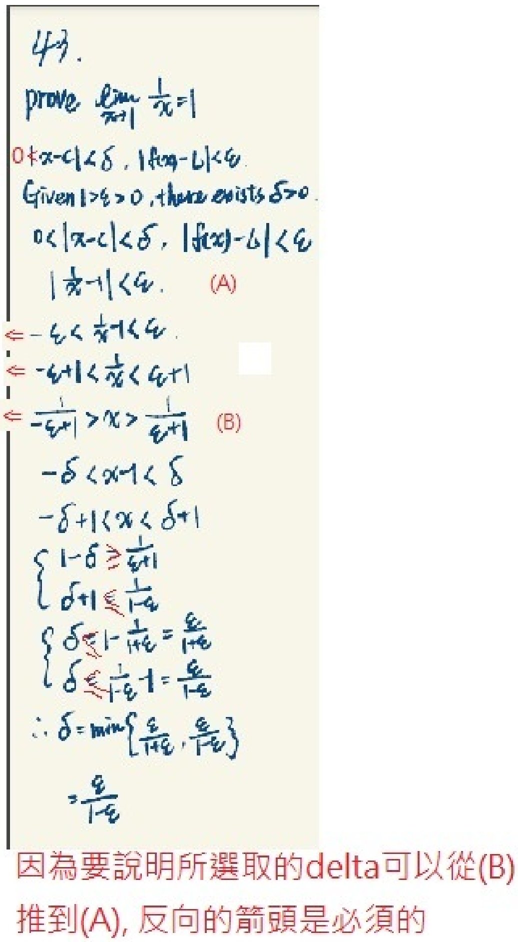


Figure 4: Section 2.3, problem 43: mistake 4

2. If $\lim\limits_{x \to c} f(x) = L$ and $\lim\limits_{x \to c} g(x) = M$

For any $\varepsilon > 0$, there exists $\delta_1 > 0$ and $\delta_2 > 0$, such that

$$0 < |x - c| < \delta_1 \Rightarrow |f(x) - L| < \frac{\varepsilon}{6}$$

$$0 < |x - c| < \delta_2 \Rightarrow |g(x) - M| < \frac{\varepsilon}{6}$$

$$\Leftrightarrow \begin{cases} -\frac{2}{3}\varepsilon < 4f(x) - 4L < \frac{2}{3}\varepsilon \\ -\frac{1}{3}\varepsilon < -2g(x) + 2M < \frac{1}{3}\varepsilon \end{cases}$$

take $\delta = \min(\delta_1, \delta_2)$

$$0 < |x - c| < \delta \Rightarrow -\varepsilon < 4f(x) - 2g(x) - (4L - 2M) < \varepsilon$$

$$\Leftrightarrow |4f(x) - 2g(x) - (4L - 2M)| < \varepsilon$$

$$\Rightarrow \lim_{x \to c} \left(4f(x) - 2g(x)\right) = 4L - 2M$$

Figure 5: Answer to homework week 03, problem 2