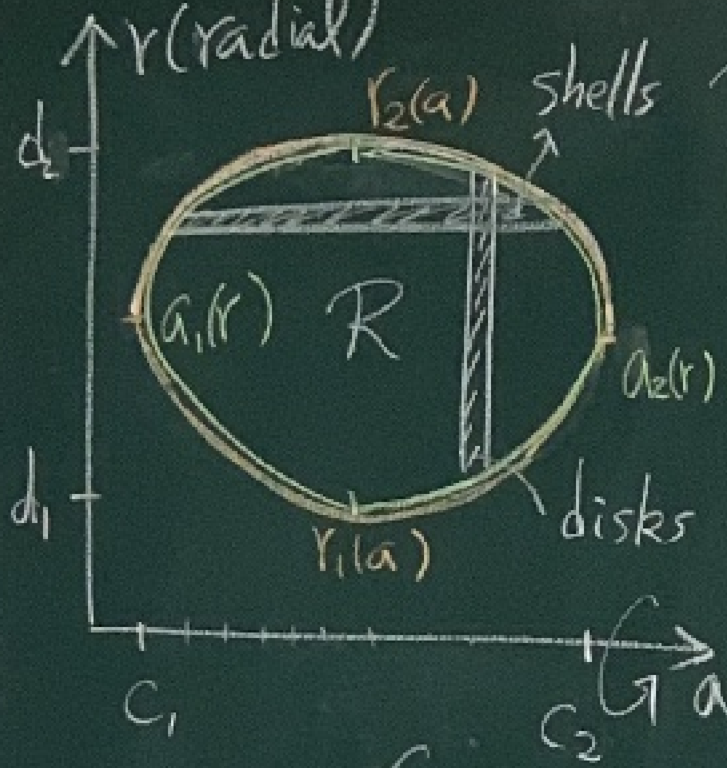


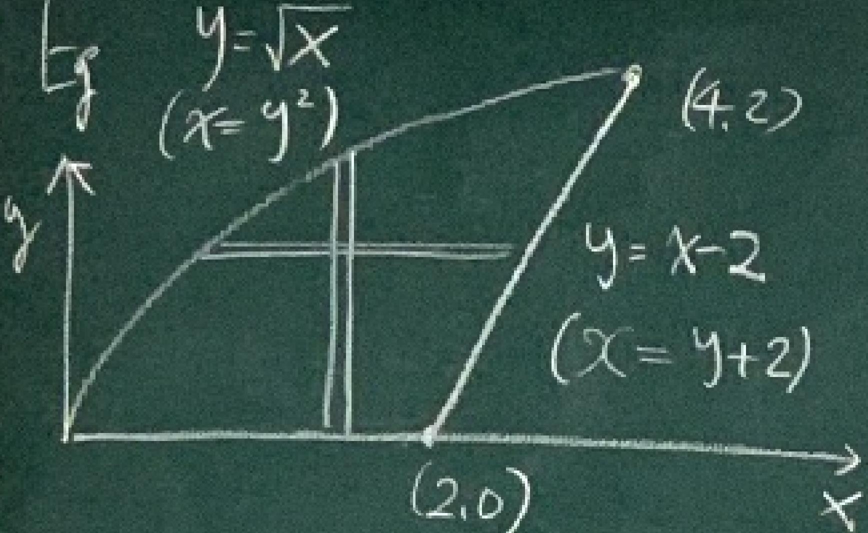
$$R=\left\{\begin{matrix} c_1 \le a \le c_2 \\ 0 \le r_1(a) \le r \le r_2(a) \end{matrix}\right\}$$

$$=\left\{\begin{matrix} d_1 \le r \le d_2 \\ 0 \le a_1(r) \le a \le a_2(r) \end{matrix}\right\}$$

$$V \overset{\text{disk}}{=} \int_{c_1}^{c_2} \pi\left(\underbrace{r_2^2(a)}_{\text{大圓}}-\underbrace{r_1^2(a)}_{\text{小圓}}\right)\underbrace{da}_{\text{厚}}$$

$$\overset{\text{shell}}{=} \int_{d_1}^{d_2} \underbrace{\left(a_2(r)-a_1(r)\right)}_{\text{高}}\underbrace{2\pi r}_{\text{圓周長}}\underbrace{dr}_{\text{厚}}$$

Eg



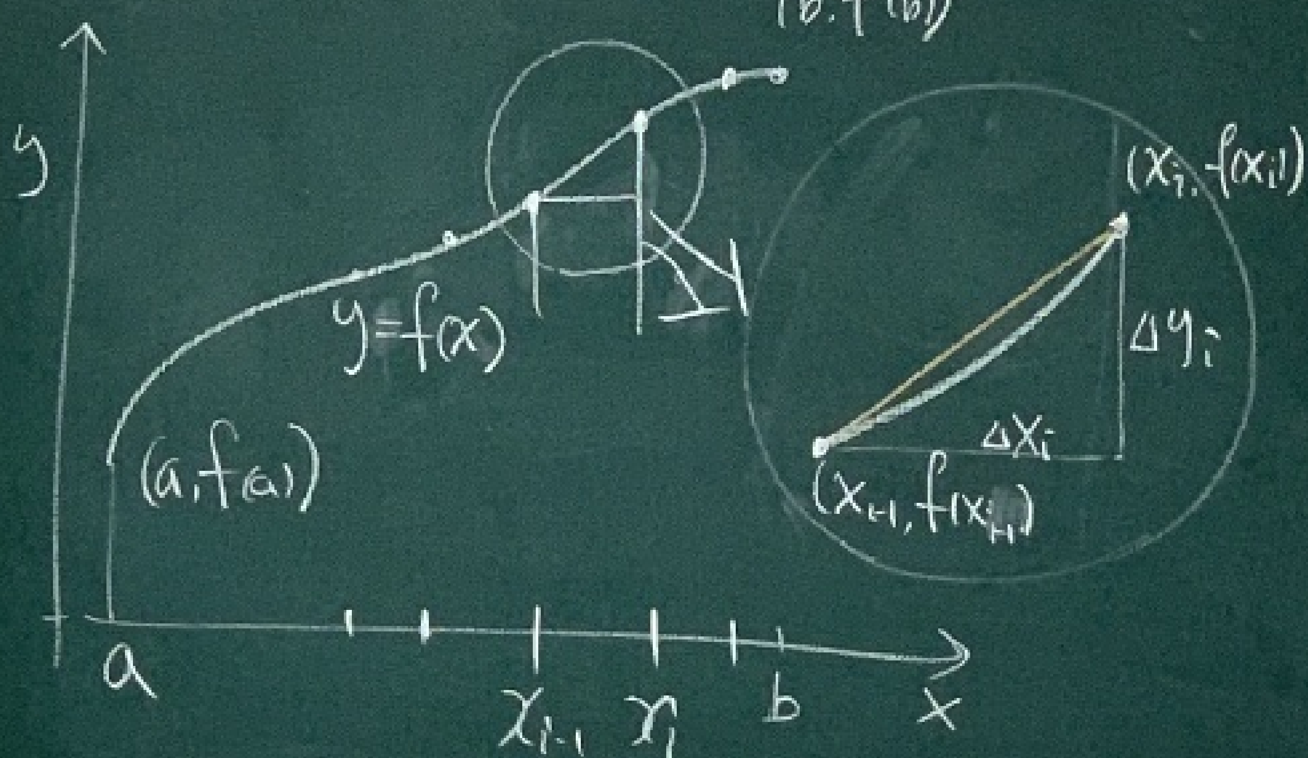
$$V \overset{\text{shells}}{=} \int_0^2 (y+2-y^2)\,2\pi y\,dy$$

$$\overset{\text{disks}}{=} \int_0^2 \pi \sqrt{x}^2\,dx + \int_2^4 \pi\left(\sqrt{x}^2-(x-2)^2\right)dx$$

$$V \overset{\text{shells}}{=} \int_0^2 (\sqrt{x}-0)\,2\pi x\,dx + \int_2^4 (\sqrt{x}-(x-2))\,2\pi x\,dx$$

$$\overset{\text{disks}}{=} \int_0^2 \pi\left((y+2)^2-(y^2)^2\right)dy$$

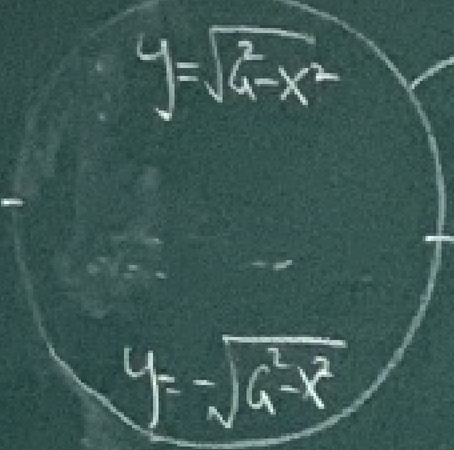
Arclength



$$L_i \cong \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2} = \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2}\,\Delta x_i$$

$$L = \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2}\,\Delta x_i = \int_{x=a}^{b} \sqrt{1 + (f'(x))^2}\, dx$$

Eg: Perimeter of circle $x^2+y^2=a^2$



$\to y=\sqrt{a^2-x^2}$, $y' = \dfrac{-x}{\sqrt{a^2-x^2}}$

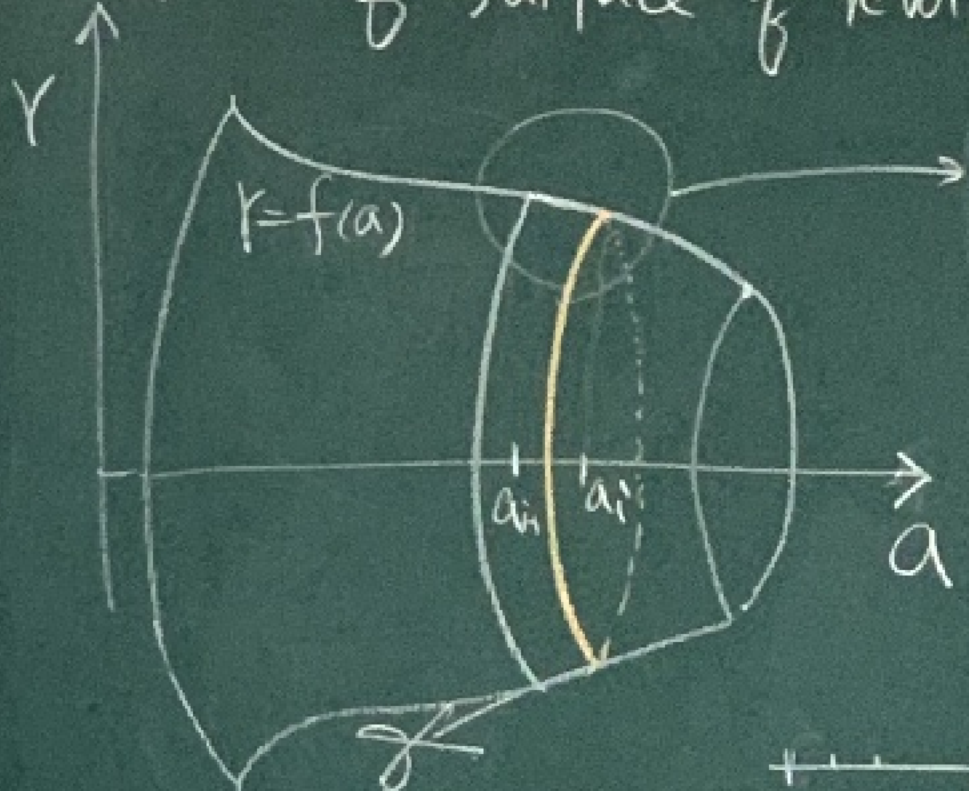
$$L = \underset{\text{上+下}}{2}\int_{-a}^{a}\sqrt{1+\frac{x^2}{a^2-x^2}}\,dx$$

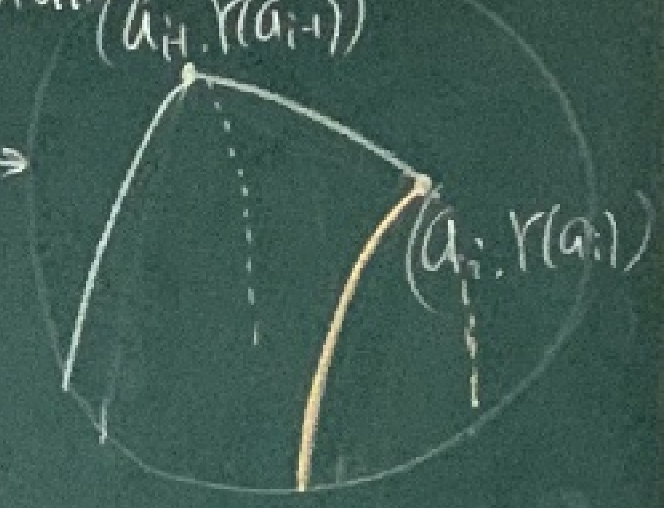
$$=2\int_{-a}^{a}\frac{a}{\sqrt{a^2-x^2}}\,dx = 2a\int_{-a}^{a}\frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\,d\frac{x}{a} = 2\pi a$$

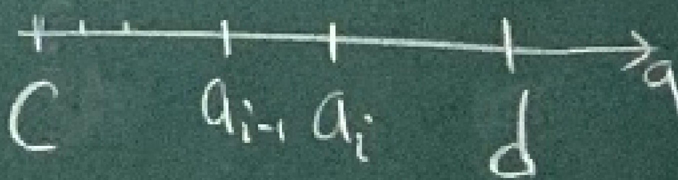
Remark: If the curve is given by $x=g(y)$

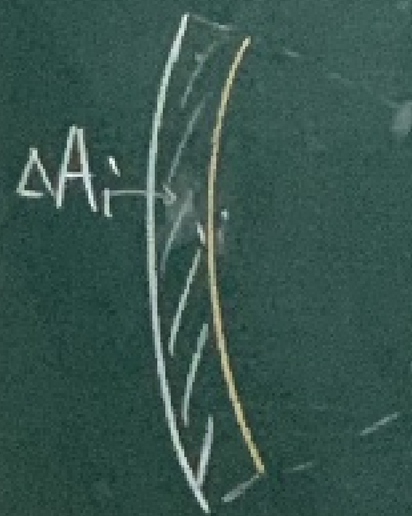
then $L=\int_c^d\sqrt{1+(g'(y))^2}\,dy$

Area of surface of revolution









$$\Delta A_i = 2\pi f(a_i)\sqrt{(\Delta a_i)^2 + (\Delta f_i)^2}$$

$$A = \lim_{n\to\infty}\sum_{i=1}^{n} 2\pi f(a_i)\sqrt{1+\left(\frac{\Delta f_i}{\Delta a_i}\right)^2}\,\Delta a_i$$

$$= \int_c^d 2\pi f(a)\sqrt{1+(f'(a))^2}\,da$$

Surface area of a donut

$$(x-R)^2 + y^2 = r^2$$

$$x = R \pm \sqrt{r^2 - y^2}$$

$$x' = \mp \frac{y}{\sqrt{r^2 - y^2}}$$

$$A_{\updownarrow} \overset{?}{=} \int_{-r}^{r} 2\pi\left(R + \sqrt{r^2-y^2}\right)\sqrt{1 + \frac{y^2}{r^2-y^2}}\, dy$$

$$+ \int_{-r}^{r} 2\pi\left(R - \sqrt{r^2-y^2}\right)\sqrt{1 + \frac{y^2}{r^2-y^2}}\, dy$$

$$= \int_{-r}^{r} 4\pi R \frac{r}{\sqrt{r^2-y^2}}\, dy = (2\pi R)(2\pi r)$$