The substitution Rule

If $u = g(x)$ is diff. then

$$\int f(g(x))\, g'(x)\, dx = \int f(u)\, du$$

$$= F(g(x)) + C$$

where $F' = f$.

$$\int f(g(x)) \frac{dg}{dx} dx = \int f(g(x))\, dg = F(g(x)) + C$$

Eg: $\displaystyle\int \frac{\cos x}{1+\sin^2 x} dx = \int \frac{d\sin x}{1+\sin^2 x} = \tan^{-1}(\sin x) + C$

Eg $\int \sec x\,dx$

$$= \int \frac{1}{\cos x}\,dx = \int \frac{\cos x}{\cos^2 x}\,dx$$

$$= \int \frac{d\sin x}{1-\sin^2 x} = \int \frac{ds}{1-s^2}$$

$$= \frac{1}{2}\int \left(\frac{1}{s+1} - \frac{1}{s-1}\right) ds$$

$$= \frac{1}{2}\left(\ln|s+1| - \ln|s-1|\right) + C = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + C$$

Eg) $\int x\sqrt{2x+1}\,dx$

let $u=2x+1$, $du=2\,dx$

$$=\int \frac{u-1}{2}\sqrt{u}\,\frac{du}{2} = \frac{u^{\frac{5}{2}}}{10}-\frac{u^{\frac{3}{2}}}{6}+C$$

or $v=\sqrt{2x+1}$, $v^2=2x+1$, $2v\,dv=2\,dx$

$$=\int \frac{v^2-1}{2}\,v\cdot v\,dv = \frac{v^5}{10}-\frac{v^3}{6}+C$$

$$=\frac{(2x+1)^{\frac{5}{2}}}{10}-\frac{(2x+1)^{\frac{3}{2}}}{6}+C$$

Eg $\displaystyle\int \frac{2z\,dz}{\sqrt[3]{z^2+1}} = \int \frac{dz^2}{(z^2+1)^{\frac{1}{3}}}$

$\displaystyle = \int (z^2+1)^{\frac{-1}{3}}\, d(z^2+1) = \frac{3}{2}(z^2+1)^{\frac{2}{3}} + C$

Eg $\displaystyle\int \sin^2 x\, dx$

$\displaystyle = \frac{1}{2}\int (1-\cos 2x)\,dx$

$\displaystyle = \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C$

$$\int \frac{\cos\sqrt{\theta}}{\sqrt{\theta}\,\sin\sqrt{\theta}}\,d\theta$$

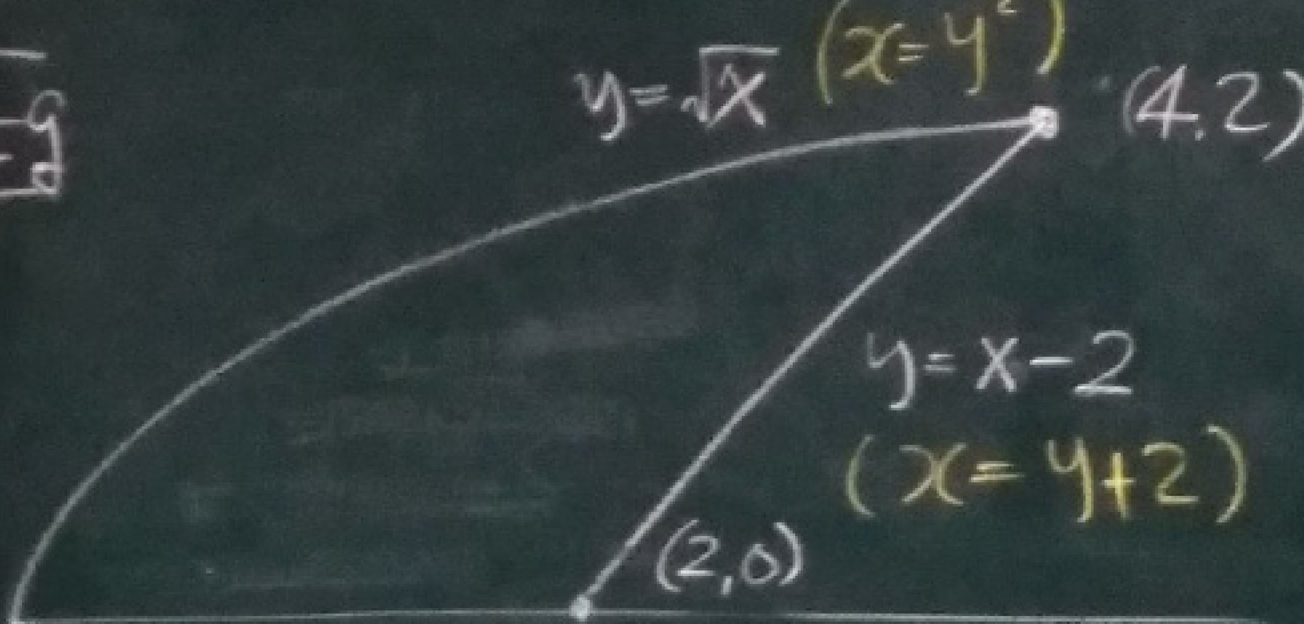
$$d\sqrt{\theta} = \frac{1}{2}\frac{1}{\sqrt{\theta}}\,d\theta.$$

$$= 2\int \frac{\cos\sqrt{\theta}\;d\sqrt{\theta}}{\sin\sqrt{\theta}}$$

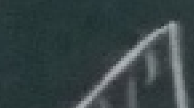
$$= 2\int \frac{d\sin\sqrt{\theta}}{\sin\sqrt{\theta}} = 2\ln\left|\sin\sqrt{\theta}\right| + C$$

Area between curves

Eg



A =  +  (I)

or  −  (II) **or**  (III)

$$\text{I} = \int_0^2 \sqrt{x}\,dx + \int_2^4 \overset{(上-下)}{\sqrt{x}-(x-2)}\,dx$$

$$= \frac{2}{3}x^{\frac{3}{2}}\Big|_0^2 + \left(\frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{2} + 2x\right)\Big|_2^4$$

$$= \frac{10}{3}$$

$$\text{II} = \int_0^4 \sqrt{x}\,dx - 2 = \frac{2}{3}x^{\frac{3}{2}}\Big|_0^4 - 2 = \frac{10}{3}$$

$$\text{III} = \int_{y=0}^{2} \underset{(右-左)}{(y+2-y^2)}\,dy = \frac{y^2}{2} + 2y - \frac{y^3}{3}\Big|_0^2$$

$$= \frac{10}{3}$$