

# The substitution Rule

If  $u = g(x)$  is diff. then

$$\int f(g(x)) g'(x) dx = \int f(u) du = F(g(x)) + C$$

where  $F' = f$ .

$$\int f(g(x)) \frac{dg}{dx} dx = \int f(g(x)) dg = F(g(x)) + C$$

Eg:  $\int \frac{\cos x}{1 + \sin^2 x} dx = \int \frac{d \sin x}{1 + \sin^2 x} = \tan^{-1}(\sin x) + C$

$$\text{Eg } \int \sec x dx$$

$$= \int \frac{1}{\cos x} dx = \int \frac{\cos x}{\cos^2 x} dx$$

$$= \int \frac{d \sin x}{1 - \sin^2 x} = \int \frac{ds}{1 - s^2}$$

$$= \frac{1}{2} \int \left( \frac{1}{s+1} - \frac{1}{s-1} \right) ds$$

$$= \frac{1}{2} \left( \ln |s+1| - \ln |s-1| \right) + C = \ln \sqrt{\frac{1 + \sin x}{1 - \sin x}} + C$$

$$\text{Ex 9) } \int x \sqrt{2x+1} dx$$

$$\text{let } u=2x+1, \quad du=2 dx$$

$$= \int \frac{u-1}{2} \sqrt{u} \frac{du}{2} = \frac{u^{\frac{5}{2}}}{10} - \frac{u^{\frac{3}{2}}}{6} + C$$

$$\text{or } V = \sqrt{2x+1}, \quad V^2 = 2x+1, \quad 2V dV = 2 dx$$

$$= \int \frac{V^2-1}{2} V \cdot V dV = \frac{V^5}{10} - \frac{V^3}{6} + C$$
$$= \frac{(2x+1)^{\frac{5}{2}}}{10} - \frac{(2x+1)^{\frac{3}{2}}}{6} + C$$

$$\begin{aligned}\text{Eg } \int \frac{2z dz}{\sqrt[3]{z^2+1}} &= \int \frac{dz^2}{(z^2+1)^{\frac{1}{3}}} \\ &= \int (z^2+1)^{-\frac{1}{3}} d(z^2+1) = \frac{3}{2}(z^2+1)^{\frac{2}{3}} + C\end{aligned}$$

$$\begin{aligned}\text{Eg } \int \sin^2 x dx \\ &= \frac{1}{2} \int (1 - \cos 2x) dx \\ &= \frac{1}{2} \left( x - \frac{1}{2} \sin 2x \right) + C\end{aligned}$$

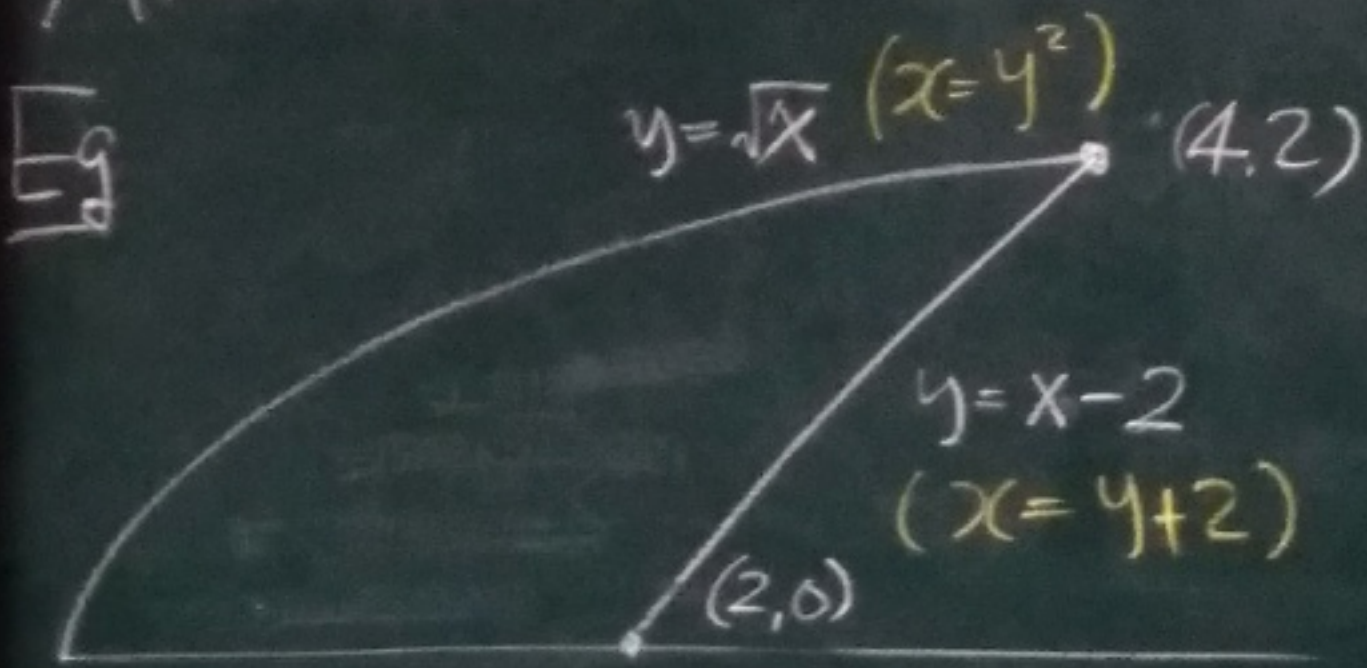
$$\int \frac{\cos \sqrt{\theta}}{\sqrt{\theta} \sin \sqrt{\theta}} d\theta$$

$$d\sqrt{\theta} = \frac{1}{2} \frac{1}{\sqrt{\theta}} d\theta$$

$$= 2 \int \frac{\cos \sqrt{\theta} d\sqrt{\theta}}{\sin \sqrt{\theta}}$$

$$= 2 \int \frac{d \sin \sqrt{\theta}}{\sin \sqrt{\theta}} = 2 \ln |\sin \sqrt{\theta}| + C$$

# Area between curves



$$A = \text{[shaded area under } y = \sqrt{x} \text{ from } x=0 \text{ to } x=4\text{]} + \text{[shaded area under } y = x - 2 \text{ from } x=2 \text{ to } x=4\text{]} \quad (\text{I})$$

$$\equiv \text{[shaded area under } y = \sqrt{x} \text{ from } x=0 \text{ to } x=4\text{]} - \text{[shaded area under } y = x - 2 \text{ from } x=0 \text{ to } x=2\text{]} \quad (\text{II}) \equiv \text{[shaded area under } y = \sqrt{x} \text{ from } x=0 \text{ to } x=4\text{]} \quad (\text{III})$$

$$\begin{aligned}
 I &= \int_0^2 \sqrt{x} dx + \int_2^4 (\sqrt{x} - (x-2)) dx \\
 &= \frac{2}{3} X^{\frac{3}{2}} \Big|_0^2 + \left( \frac{2}{3} X^{\frac{3}{2}} - \frac{x^2}{2} + 2x \right) \Big|_2^4 \\
 &= \frac{10}{3}
 \end{aligned}$$

$$II = \int_0^4 \sqrt{x} dx - 2 = \frac{2}{3} X^{\frac{3}{2}} \Big|_0^4 - 2 = \frac{10}{3}$$

$$\begin{aligned}
 III &= \int_{y=0}^2 (y+2-y^2) dy = \frac{y^2}{2} + 2y - \frac{y^3}{3} \Big|_0^2 \\
 &= \frac{10}{3}
 \end{aligned}$$