an interval

Def Let $D \subseteq \mathbb{R}$ be an interval

$f: D \to \mathbb{R}$, $c \in D$ is a

abs max
(local max)
abs min
((local min))

if $f(x) \leq f(c)$
$\geq$

for all $x \in D$ $((c-\delta, c+\delta) \cap D)$

$\delta > 0$

Note: this contains the def. on p237 and p239 (+5 lines after it)

Eg. $f : \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) \mapsto \mathbb{R}$. $f(x) = \tan x$.

Then f does not have abs. $\begin{matrix}\text{min}\\ \text{max}\end{matrix}$

Eg: $f : \overset{D}{\overset{\shortparallel}{(0,\infty)}} \mapsto \mathbb{R}$

$$f(x) = \frac{1}{x}$$

has no abs $\begin{matrix}\text{max}\\ \text{min}\end{matrix}$ on D

__Thm 1__: If f is cont. on $[a,b]$. Then there exist $x_1, x_2 \in [a,b]$ such that

$$f(x_1) = m, \quad f(x_2) = M$$

are abs. min and abs max.

i.e. $m \le f(x) \le M$ for all $x \in [a,b]$

Thm 2 $f : [a,b] \xrightarrow{\text{cont.}} \mathbb{R}$

If f has a local $\begin{matrix}\text{min}\\ \text{max}\end{matrix}$ at $c \in (a,b)$ and f is diff at c, $\substack{\Longrightarrow \\ \not\Longleftarrow}$ $f'(c) = 0$

Eg: $f : [-1,1] \longrightarrow \mathbb{R}$

$f(x) = x^3$, $f'(0) = 0$.

<u>Pf</u>: If f has a local min at c.

$$\Rightarrow f'(c)=\lim_{x\to c^+}\frac{f(x)-f(c)}{x-c}\ \frac{(\text{"}+\text{"})}{(\text{"}+\text{"})_{x>c}} \geq 0$$

$$=\lim_{x\to c^-}\frac{f(x)-f(c)}{x-c}\ \frac{(\text{"}+\text{"})}{(\text{"}-\text{"})_{x<c}} \leq 0$$

$$\therefore\ f'(c)=0$$

Similarly for local max.

From Thm 2, possible local $\begin{matrix}\text{min}\\ \text{max}\end{matrix}$ include: (assume f is cont. on $[a,b]$)

critical points
(i) $c \in (a,b)$, $f'(c) = 0$
(ii) $c \in (a,b)$, $f'(c)$ not exist.
(iii) $c = a$ or $c = b$

How to find <u>abs</u> max/min of f?

<u>Step 1</u>: find all critical points of f and end points

Step 2: compare values of f on them:

Eg4 Find abs max/min for $f(x)=x^{\frac{2}{3}}$ on $[-2,3]$

Sol critical points.

$$f'(x)=\frac{2}{3}x^{\frac{-1}{3}}$$

$f'(0)$ does not exist.

critical point

Compare $f(-2)$, $f(3)$, $f(0)$

$\Rightarrow$ abs max: $(x=3)$ $=f(3)=3^{2/3}$ abs min: $(x=0)$ $=f(0)=0$