Study Guide for Chap 16

1. Section 16.1: skip the definitions of first moments, center of mass and moments of inertia.

   Section 16.2: skip the definitions of work, circulation, flow and flux.

   All exam problems will be expressed in explicit mathematical symbols.

2. Section 16.1, 16.2:

   Study the meanings of

$$\int_C f(x,y,z)ds,$$

$$\int_C \boldsymbol{F}(x,y,z)\cdot \boldsymbol{T}ds = \int_C \boldsymbol{F}(x,y,z)\cdot d\boldsymbol{r} = \int_C M(x,y,z)\,dx + N(x,y,z)\,dy + P(x,y,z)\,dz,$$

$$\int_C \boldsymbol{F}(x,y)\cdot \boldsymbol{n}ds = \oint_C M(x,y)\,dy - N(x,y)\,dx$$

   and how to calculate them using a properly chosen parametrization of C: $\boldsymbol{r}(t)$, $t_0 \le t \le t_1$.

   A few points to think over:

   Which of them is (are) independent of the orientation of C? Which of them depend(s) on the orientation of C?

   How do you choose the parametrization $\boldsymbol{r}(t)$ so that the direction of $\boldsymbol{T}$ comply with the orientation of C?

   How is the outward normal $\boldsymbol{n}$ related to $\boldsymbol{T}$ if the parametrization of C is increasing in the counter-clockwise direction?

3. Section 16.3:

   Study and memorize the definitions of 'path independent', 'conservative' (p938) and 'potential function' (p939).

   Study the proof of Theorem 1:'Fundamental Theorem of Line Integrals' (p940).

   Study the definition of 'simply connected' (bottom of p939 and the pictures in p940).

   Study the proof of Theorem 2: 'Conservative Fields are Gradient Fields' (p941).

   Regarding the two theorems on equivalent characterization of conservative fields, Theorem 3: 'Loop Property' (p942) and 'Component test' (p943), which of them requires the domain to be simply connected? If the domain is not simply connected (roughly speaking, it has a hole), which of the implications '$\Longleftarrow$', '$\Longrightarrow$' remains valid? which one may probably be false? Is Example 5 a counter example of which case?

For given functions $M(x,y,z), N(x,y,z), P(x,y,z)$ satisfying the component test, how does one find the potential function (if it exists) by way of direct integration (example 3)?

4. Section 16.4:

   Study and memorize definitions of divergence and curl. It is easier to start with

   $$\text{div}\boldsymbol{F}(x,y,z) = \nabla \cdot \boldsymbol{F}(x,y,z), \quad \text{and} \quad \text{curl}\boldsymbol{F}(x,y,z) = \nabla \times \boldsymbol{F}(x,y,z),$$

   then reduce them to

   $$\text{div}\boldsymbol{F}(x,y) \quad \text{and} \quad \text{curl}\boldsymbol{F}(x,y,0) \cdot \boldsymbol{k}.$$

   Study and memorize Green's Theorem both in normal form (Theorem 4, p954) and in tangential form (Theorem 5, p955)

   Is Theorem 5 applicable to Example 3 of section 16.3? Which part went wrong?

5. Section 16.5, 16.6:

   Study how to calculate $\iint_S G(x,y,z)\, d\sigma$ when the surface S is given by the parametrization

   $$\boldsymbol{r}(u,v) = f(u,v)\boldsymbol{i} + g(u,v)\boldsymbol{j} + h(u,v)\boldsymbol{k}, \quad a \le u \le b, \quad c \le v \le d.$$

   Study how to calculate $\iint_S G(x,y,z)\, d\sigma$ when the surface S is given by a graph

   $$x = f(y,z), \quad (y,z) \in R,$$

   or a graph

   $$y = g(x,z), \quad (x,z) \in R,$$

   or a graph

   $$z = h(x,y), \quad (x,y) \in R.$$