Study Guide for Chap 14

1. Review and memorize definitions of limit and continuity (using ε and δ) for functions of two or more variables. Review related examples in section 14.2 and understand the reasons for the examples where the limit does not exist.

2. Study the definition of differentiability for functions of two or more variables in section 14.3. Study the relation of differentiability with the tangent plan, with linear approximations, and why existence of partial derivatives and directional derivatives at point does not imply differentiability at a point.

3. Study why differentiability at a point implies continuity at that point. Try to find a function of two variables that is continuous at a point but not differentiable there.

4. Study the relation between Chain rule and differentiability for functions of two or more variables in section 14.4. Practice the chain rule, for example, between Cartesian coordinate (x, y) and polar coordinate (r, θ) where $x = r\cos\theta$, $y = r\sin\theta$. Also study how to evaluate
$$\frac{d}{dx}\int_{g_1(x)}^{g_2(x)} f(x,t)dt.$$

5. Study the definition of the directional derivatives. Practice with some examples to compute the directional derivatives from definition.

6. Study the definition and properties of the gradient vector and its relation with tangent and normal line/plane.

7. Study the 'Second Derivative Test', how the sign of the determinant $f_{xx}f_{yy} - f_{xy}^2$ is related to 'sum of squares' and 'difference of squares', which in term determines whether a critical point is local min, local max, or neither.

8. Review the gradient analysis that helps to classify the critical points when the second derivative test is inconclusive, and how it can be applied to find absolute max and absolute min on a bounded region.

9. Review all cases for the method of the Lagrangian multipliers. For example, extreme values of
   - functions of 2 variables, with 1 constraint.
   - functions of 3 variables, with 1 constraint.
   - functions of 3 variables, with 2 constraints.

10. Review the derivation of Taylor's formula for functions of two or more variables.
11. Review partial derivatives with constrained variables. For example, partial derivatives of
    - functions of 3 variables, with 1 constraint.
    - functions of 4 variables, with 2 constraints.
    - functions of 3 variables, with 2 constraints.