Section 14.4 Prob 51.

$$F(x) = \int_a^{u(x)} g(v(x), t)\, dt$$

$$\frac{d}{dx} F(x) = ?$$

Ans:

Let $G(u, v) = \int_a^u g(v, t)\, dt$

Then $F(x) = G(u(x), v(x))$

$$F'(x) = G_u \cdot u'(x) + G_v \cdot v'(x)$$

$G_u \overset{\text{Fundamental Thm. Calculus}}{\underset{v \text{ fixed}}{=\!=\!=}} g(v, u)$

$G_v \overset{}{\underset{u \text{ fixed}}{=\!=\!=}} \int_a^u g_v(v, t)\, dt$