

## Homework Assignment for Week 11

1. Section 14.7: Problems 31, 35, 43, 44.

Hint: You can use the method of gradient analysis introduced in 190430's class. That is, plot the gradient vector  $\nabla f$  near a critical point to determine whether the critical point is a local minimum, local maximum or neither.

2. Section 14.8: Problems 1, 23, 27, 33, 35.
3. Section 14.9: Problems 7, 9, 10, 11.

Hint: An alternative method for problem 10, 11: try Taylor's formula for  $\frac{1}{1-z}$  with appropriate choice of  $z$ . For problem 11: read example 1 in page 840.

4. Section 14.9:

Taylor's formula for functions of 2 variables can be summarized as

$$f(x_0 + \Delta x, y_0 + \Delta y) = f(x_0, y_0) + \sum_{k=1}^n \frac{1}{k!} (\Delta x \partial_x + \Delta y \partial_y)^k f(x_0, y_0) \\ + \frac{1}{(n+1)!} (\Delta x \partial_x + \Delta y \partial_y)^{n+1} f(x_0 + c\Delta x, y_0 + c\Delta y), \quad 0 < c < 1.$$

(Note: the textbook uses the notations  $a, b, h, k$  in place of  $x_0, y_0, \Delta x, \Delta y$ ). Derive a similar formula for functions of 3 variables. Then use it to derive the error estimate  $|E| < \dots$  in page 816 (section 14.6).

5. Section 14.9:

Use Taylor's formula for 2 variables to derive the formula  $\Delta f = \Delta_1 + \Delta_2 + \Delta_3$  on page 5 of mse19s\_190425.pdf.