

Homework Assignment for Week 09

1. Section 14.4: Problems 29, 31, 43, 51.
2. Suppose that $F(x, y, z) = 0$ can implicitly define $x = f(y, z)$, or $y = g(z, x)$, or $z = h(x, y)$ near some point (x_0, y_0, z_0) with $F(x_0, y_0, z_0) = 0$. (for example, $F(x, y, z) = x + 2y + 3z - 4$ can). Show that, at any such point (x_0, y_0, z_0) ,

$$\frac{\partial f}{\partial y} \frac{\partial g}{\partial z} \frac{\partial h}{\partial x} = \frac{\partial f}{\partial z} \frac{\partial g}{\partial x} \frac{\partial h}{\partial y} = -1$$

3. Section 14.5: Problems 9, 15, 19, 25, 27, 29, 35, 36, 40 (See page 807).

Note: in problem 15, “direction of \mathbf{u} ” refers to a unit vector.

4. Let $f(x, y) = x^2y/(x^2 + y^2)$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$.
 - (a) Is f continuous at $(0, 0)$?
 - (b) Do f_x and f_y exist at $(0, 0)$?
 - (c) Are f_x and f_y continuous at $(0, 0)$?
 - (d) Evaluate $Df_{(\cos \theta, \sin \theta)}(0, 0)$, i.e. the directional derivative of f at $(x_0, y_0) = (0, 0)$ in the direction $(\cos \theta, \sin \theta)$, if it exists.
 - (e) Is f differentiable at $(0, 0)$?