Homework Assignment for Chapter 14

1. Section 14.2: Problems 41, 43, 49, 51, 57, 61, 63.

   Hint for problems 61, 63: Read 'Changing to Polar Coordinates' on page 781.

2. Section 14.3: Problems 19, 21, 53, 60, 65, 67, 69, 81, 91.

3. Show that if $f(x, y) = o(1) \cdot |x - x_0| + o(1) \cdot |y - y_0|$ as $(x, y) \to (x_0, y_0)$ then $f(x, y) = o(1) \cdot \sqrt{(x - x_0)^2 + (y - y_0)^2}$ and vice versa (the converse). Note that all three $o(1)$ refer to $2D$ limits as $(x, y) \to (x_0, y_0)$.

   Hint: $\sqrt{\Delta x^2 + \Delta y^2} = \frac{\Delta x}{\sqrt{\Delta x^2+\Delta y^2}}\Delta x + \frac{\Delta y}{\sqrt{\Delta x^2+\Delta y^2}}\Delta y$

4. Section 14.4: Problems 7, 29, 31, 43, 51.

5. Suppose that $F(x, y, z) = 0$ can implicitly define $x = f(y, z)$, or $y = g(z, x)$, or $z = h(x, y)$ near some point (x_0, y_0, z_0) with $F(x_0, y_0, z_0) = 0$. (for example, $F(x, y, z) = x + 2y + 3z - 4$ can). Show that, for any such point (x_0, y_0, z_0), we have

$$\frac{\partial f}{\partial y}\frac{\partial g}{\partial z}\frac{\partial h}{\partial x} = \frac{\partial f}{\partial z}\frac{\partial g}{\partial x}\frac{\partial h}{\partial y} = -1$$

6. Section 14.5: Problems 9, 15, 19, 25, 27, 29, 35, 36, 40 (See page 807).

7. Let $f(x, y) = x^2y/(x^2 + y^2)$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$.

   (a) Is f continuous at $(0, 0)$?

   (b) Do f_x and f_y exist at $(0, 0)$?

   (c) Evaluate the directional derivative of f at $(x_0, y_0) = (0, 0)$ in the direction $(\cos\theta, \sin\theta)$, i.e. $df/ds_{(0,0),(\cos\theta,\sin\theta)}$, if it exists.

   (d) Is f differentiable at $(0, 0)$?

8. Section 14.6: Problems 1, 9, 13, 19, 25, 33, 39, 45, 59, 63, 66, 67.

   Section 14.7: Problems 1, 19, 31, 35, 39, 43, 44, 49, 51, 55, 61.

   Hint for problem 44: Do this problem using the gradient analysis. That is, plot ∇f near the critical point to determine whether the critical point is a local minimum, local maximum or neither.

9. Section 14.8: Problems 1, 23, 27, 33, 35.

10. Section 14.9: Problems 7, 9, 11.

11. Taylor's formula for functions of 2 variables can be summarized as

$$f(x_0+\Delta x, y_0+\Delta y) = f(x_0,y_0) + \sum_{k=1}^{n} \frac{1}{k!}\Big(\Delta x\partial_x + \Delta y\partial_y\Big)^k f(x_0,y_0)$$

$$+\frac{1}{(n+1)!}\Big(\Delta x\partial_x + \Delta y\partial_y\Big)^{n+1} f(x_0+c\Delta x, y_0+c\Delta y), \quad 0<c<1.$$

Derive a similar formula for functions of 3 variables.

12. Section 14.10: Problems 3, 7, 9, 12.

13. Continue on problem 12: Give a formula for $\left(\frac{\partial u}{\partial x}\right)_y$ where $u=u(x,y,z,w)$ and $f(x,y,z,w)=0$, $g(x,y,z,w)=0$.

14. Follow up on example 5 of section 14.8: Let $f(x,y,z)=x^2+y^2+z^2$ and compute $\frac{df}{dx}$ at $P_1=(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2},1-\sqrt{2})$, subject to the two constraints $g_1(x,y,z)=x^2+y^2-1=0$ and $g_2(x,y,z)=x+y+z-1=0$. That is, evaluate $\frac{d}{dx}f(x,y(x),z(x))$ at $x=\frac{\sqrt{2}}{2}$, where $y=y(x)$ and $z=z(x)$ are implicitly given by the constraints $g_1=0$ and $g_2=0$. Can you explain why you get $\frac{df}{dx}=0$ there?