Final Exam

June 20, 2013, 10:10 AM.
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1. (10 pts) Let $D = \{1 \le x \le 1,\ 0 \le xy \le 1,\ 0 \le z \le 1\}$. Evaluate
$$\int\int\int_D (x^2y + xyz)dV.$$

2. (10 pts) Find the work done by $\boldsymbol{F}(x, y, z) = (xy, yz, x^2)$ over the line segment joining from $(2, 2, 2)$ to $(1, 1, 1)$.

3. (10 pts) Integrate $g(x, y, z) = 2x - y + z$ over the portion of the plane $x + y + z = 1$ that lies in the first octant.

4. (15 pts) Let $R = \{1 \le x^2 + y^2 \le 4\}$, $\boldsymbol{F}(x, y) = (2y, x)$, $\boldsymbol{G}(x, y) = (x, y)$, $\boldsymbol{H}(x, y) = (\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2})$.

   Which one(s) of $\boldsymbol{F}$, $\boldsymbol{G}$ and $\boldsymbol{H}$ is (are) conservative on R? (That is, which one(s) of $\int \boldsymbol{F} \cdot d\boldsymbol{r}$, $\int \boldsymbol{G} \cdot d\boldsymbol{r}$ and $\int \boldsymbol{H} \cdot d\boldsymbol{r}$ is (are) zero on every closed loop in R?) Explain.

5. (40 pts) Let $D = \{x^2 + y^2 + z^2 < 4, x > 0\}$, $S = \{x^2 + y^2 + z^2 = 1, z > 0\}$ and $\boldsymbol{F}(x, y, z) = (x + z, xz, xy)$.

   (a) Which one(s) of S and D does Stokes Theorem apply? Verify Stokes Theorem on it (them). That is, compute integrals on both sides of the Stokes Theorem and check they are the same.

   (b) Do the same for divergence Theorem.

6. (15 pts) Let $\boldsymbol{F}(x, y, z) = \dfrac{(x, y, z)}{\left(\sqrt{x^2 + y^2 + z^2}\right)^3}$. Evaluate
$$\int\int_{\frac{(x-1)^2}{4}+y^2+z^2=1} \boldsymbol{F} \cdot \boldsymbol{n}\ d\sigma.$$