Calculus II, Spring 2016

Quiz 3

Apr 14, 2016

1. Give formal definitions of (1): $\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$, and (2): The function $z = f(x,y)$ is differentiable at (x_0, y_0).
2. Does $\lim_{(x,y)\to(0,0)} \dfrac{x^3 - xy^2}{x^2 + y^2}$ exist? Explain.
3. Let $f(x,y) = \dfrac{xy^2}{x^2 + y^4}$ for $(x,y) \neq (0,0)$ and $f(0,0) = 0$. Do $f_x(0,0)$ and $f_y(0,0)$ exist? Explain. Is f continuous at $(0,0)$? Explain.
4. Let $F(x) = \displaystyle\int_0^x \cos(x^2 - t^2)dt$. Evaluate $F'(0)$. Give details.
5. Let $f(x,y) = x^2 - xy + 2y^2$. Find the direction $\boldsymbol{u}$ (a unit vector) for which the directional derivative $\left(\dfrac{df}{ds}\right)_{\boldsymbol{u},(1,1)}$ (that is, $D_{\boldsymbol{u}} f(1,1)$), is largest.

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