Calculus II, Spring 2016

Quiz 2

Mar 17, 2016

Show all details.

1. State (need not prove) the Leibnitz test for alternating series. Then give an example of a_n such that $\sum_n a_n$ converges and $\sum_n |a_n|$ diverges.

2. Give an example of a power series that converges on $[0, 2]$ and diverges elsewhere. Explain. Do the same for $(3, 7]$.

3. Give the power series representation (centered at $a = 0$) of $\frac{1}{1+x}$ and find its radius of convergence. Then use it to find the power series representation (centered at $a = 0$) of $\ln(1 + x)$.

4. Suppose we know the power series representation of $\sin x$ (centered at $a = 0$), $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ is valid for all $x \in \mathbb{R}$. Use it to find the power series representation of $\cos x$ (centered at $a = 0$). Then use them to find the first few terms of Taylor series generated by $\tan x$ (centered at $a = 0$), upto and including the coefficient of x^5.

5. True or False? Explain.

   If $f(x) = \sum_{n=0}^{\infty} a_n(x - a)^n$ converges on $(-1, 1)$, then $a_n = \frac{f^{(n)}(a)}{n!}$.

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