Calculus II, Spring 2004

# Quiz 4

May 26 2004

Show all details.

1. Compute the volume of an ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1$.

2. Compute $\int_0^\infty e^{-\frac{x^2}{2}} dx$. (Hint: transform into a double integral and use polar coordinates)

3. State Green's Theorem in both circulation and flux forms.

4. Compute the area enclosed by $x = \cos^3 t$, $y = \sin^3 t$, $0 \leq t \leq 2\pi$.
(Hint: check $\int \pm x dy \pm y dx$)

5. Change the integral $\int_0^{2\pi} \int_0^{\sqrt{2}} \int_r^{\sqrt{4-r^2}} r dz dr d\theta$ into integrals in Cartesian ($dzdxdy$) and spherical coordinates. Pay attention for the limits of integration in new coordinates.

6. Compute $\int_C 2xdx - y^2 dy + \frac{1}{1+z^2} dz$ where $C$ is given by $x(t) = \cos 2\pi t$, $y(t) = \sin 2\pi\sqrt{t}$, $z(t) = x(t) - 2y(t)$, $0 \leq t \leq 1$. (Hint: something special about $(2x, -y^2, \frac{1}{1+z^2})$)

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