Midterm Exam

April 15 2004, 10:10 AM. Show all details. Total 125 points, maximum 100 points.

1. (10 pts) Is $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$. convergent for $p = 1/2$? How about $p = 1$? Explain.

2. (5+10 pts)

   (a) State (need not prove) the Leibniz's Theorem for alternating series (A condition for an alternating series to converge).

   (b) Give an example of a series that converges conditionally. Explain.

3. (10 pts) Find a power series that converges on $[1, 3]$ and diverges elsewhere. Explain.

4. (10 pts) Write down a_0, a_1, a_2 for the series expansion of $y(x)$ around $x = 0$. Where $y(x)$ is given by
$$\begin{aligned} dy/dx &= y - y^2 \\ y(0) &= 1 \end{aligned} \tag{1}$$

5. (10 pts) State (need not prove) Taylor's Theorem (You can ignore the assumptions on the function and simply write down n terms approximation and the formula of the remainder term).

6. (5+10 pts)

   (a) Write down the Taylor series of $\ln(x)$ around $x = 1$ (you don't need to show that $\ln(x) =$ the Taylor series, just write down the series) and give the radius of convergence. Explain.

   (b) Find the sum of the series $1 - \frac{\pi^2}{4 \cdot 2!} + \frac{\pi^4}{16 \cdot 4!} - \cdots + (-1)^n \frac{\pi^{2n}}{2^{2n} \cdot (2n)!} + \cdots$.

7. (10 pts) Let a parametrized curve be given by $x = \cos t$, $y = t + \sin t$.

   (a) Find the slope of the curve at $t = \pi/4$.

   (b) Find the length of the curve on $0 \leq t \leq \pi/2$. Give your answer as a definite integral, but need not find the numerical value of the integral.

8. (5+5+5 pts) In each of the following, the coordinate in one of the 3 coordinate systems: rectangular – (x, y, z), cylindrical – (r, θ, z) and spherical – (ρ, φ, θ) ($0 \leq \varphi \leq \pi$, $0 \leq \theta < 2\pi$) is given. Write down the coordinates in the other two coordinate systems.

   (a) $(1, 2, 3)$ in rectangular coordinate system.

   (b) $(1, 2, 3)$ in cylindrical coordinate system.

   (c) $(1, 2, 3)$ in spherical coordinate system.

9. (20 pts) Find $\boldsymbol{T}$, $\boldsymbol{N}$, $\boldsymbol{B}$, κ and τ for the curve $x = \cos t$, $y = \sin t$, $z = 2t$.
   Show details of your work.

10. (10 pts) Find the linearization $L(x, y)$ for $f(x, y) = xy^2 + \sin(x + y)$ at $P = (1, 1)$.