Midterm Exam

Nov 06 2003, 10:10 AM - 12:00 AM

Show all details.

(1) (10 pts) Write down the definition of

$$\lim_{x\to 1}\frac{1}{x}=1$$

in terms of ϵ and δ. Then find a δ that works for $\epsilon = 0.01$.

(2) (10 pts) Find an approximate value of $\sqrt[4]{1.008}$ using linear approximation. Show that the error of the approximation is smaller than 10^{-4}.

(3) (10 pts) Find the derivatives of following functions:

(a): $y = \ln(\cosh x)$

(b): $y = (\frac{\sin^{-1} x}{1+x})^3$

(4) (10 pts) Use implicit differentiation to find $y'(x)$ and $y''(x)$ at $(1,1)$ where $y(x)$ is implicitly given by $x^4 + y^2 = 2$.

(5) (10 pts) Find the tangent and normal line of $y = \sinh^{-1} x$ at $(1, \sinh^{-1} 1)$.

(6) (15 pts) Graph $y = xe^{-x^2/2}$. Indicate all the asymptotes, critical points and inflection points. Determine whether the critical points are local extrema.

(7) (15 pts) Find the limits of the following expressions, distinguish between the cases when the limit is ∞ or $-\infty$ and the limit does not exist.

(a): $\lim_{x\to 0^+} x^{\ln(1-x)}$

(b): $\lim_{x\to 0} \frac{x^2 \sin\frac{1}{x}}{\sin^{-1} x}$

(8) (20 pts) Let $f(x) = e^{x^2}$ defined only on $x \geq 0$.

(a): Find the inverse function of f, indicate the domain of definition and compute its derivative $(f^{-1})'$.

(b): Determine whether the following statement is true or false. If false, point out and correct the errors:

$$\frac{df^{-1}(x)}{dx} = \frac{1}{\frac{df}{dx}} = \frac{1}{2xe^{x^2}} = \frac{1}{2x}e^{-x^2}$$