Final Exam

Jan 08 2004, 10:10 AM - 12:40 AM
Show all your work

(1) (10 pts) Compute $\frac{d}{dx}\int_x^{2x} e^{\sqrt{y^2+1}}dy$.

(2) (10 pts) Find

$$\lim_{n\to\infty}\sum_{i=1}^{n}\exp(\frac{i}{n})\cdot\frac{1}{n}$$

(3) (15 pts) Suppose $f(x)$ is continuous, $f(1) = f(2) = 0$ and $f(x) > 0$ on $1 < x < 2$. Write down the formula for the following quantities (formula only, no numerical values required).

(a): The area and perimeter of the region enclosed by $y = f(x)$ and the x axis.

(b): The volume and surface area obtained by rotating (a) around the x axis.

(c): The volume and surface area obtained by rotating (a) around the y axis.

(4) (10 pts) Which of the following improper integrals converge and which of them diverge? Explain.

(a): $\int_1^\infty \frac{1}{\sqrt{x^2+x}}dx$

(b): $\int_1^\infty \frac{\ln x}{x^2}dx$

(5) (10 pts each) Find the following integrals.

(a): $\int \frac{\cos x}{1+\sin(x)}dx$

(b): $\int \frac{1}{2-\sin x}dx$

(c): $\int \frac{3+x^2}{1-x^4}dx$

(d): $\int_{-1}^1 \frac{1}{x^2+2x+2}dx$.

(e): $\int_0^\infty x^2e^{-x}dx$

(f): $\int_0^2 \frac{1}{(4+x^2)^2}dx$