Calculus II, Spring 2016

Midterm 2

May 10, 2016

Show all details.

1. True or False?

   If $f(x, y)$ is differentiable at $(0, 0)$, then $f(x, y)$ is continuous at $(0, 0)$.

2. Evaluate
$$\frac{d}{dy}\int_1^{2+y^2} \frac{\cos(xy)}{x} dx.$$

3. Find the equation of plane normal to the following curve at $(1, -1, 1)$
$$\begin{cases} x^2 + 2y^2 + 3z^2 = 6 \\ x + y + z = 1 \end{cases}$$

4. Show that, for all $a \in \mathbb{R}$, the point $(x_0, y_0) = (0, 0)$ is a critical point of the function
$$f_a(x, y) = (a - 1)(x + y)^2 + (a + 1)(x - 2y)^2.$$
   For what values of a is the point $(0, 0)$ a local minimum, a local maximum and a saddle point, respectively?

5. Find absolute maxima and minima of $f(x, y) = x^2 + xy + y^2 - 6x + 2$ on the rectangular $0 \le x \le 5$, $-3 \le y \le 3$.

6. Give an example of a constraint optimization problem that, upon applying the method of Lagrangian multipliers, results in a system of 5 equations with 5 unknowns (and write down the equations). Need not solve it.

7. Use Taylor's formula to find the quadratic approximation of $f(x, y, z) = \dfrac{1}{1 - x - y + z}$ near the origin.

8. Evaluate $\left(\frac{\partial u}{\partial x}\right)_y$ at $(x, y, z, w) = (1, 1, 1, 1)$ where $u(x, y, z, w) = x^2 + y^2 + z^2 + w^2$ with the constraint $x + y + z + w = 4$ and $x - y + z - w = 0$.

9. Evaluate
$$\int_0^2 \int_y^2 x^2 \cos(xy)\, dxdy$$

10. Change
$$\int_{\sqrt{2}}^2 \int_{\sqrt{4-y^2}}^y dxdy$$
    into an equivalent polar integral and evaluate the polar integral.

11. Is $\int_1^\infty \sin \frac{1}{x^2}$ convergent? Explain.

12. Evaluate $\int_{n=0}^\infty \frac{x^n}{n+2}$ on $|x| < 1$ using computational rules of power series.