Final Exam

Jun 16, 2016

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1. (10 pts) Replace
$$\int_0^{2\pi}\int_0^1\int_0^{\sqrt{2-r^2}} rdz\ dr\ d\theta$$
by triple integrals in spherical coordinates in the order $d\rho d\phi d\theta$ and $d\phi d\rho d\theta$, respectively. Need not evaluate them.

2. (15 pts) Evaluate the surface area of $S = \{z = \sqrt{x^2+y^2},\ 1 \le xy \le 2,\ 1 \le x/y \le 3,\ x > 0,\ y > 0\}$.

3. Let $\boldsymbol{F}(x, y) = (M(x, y), N(x, y))$ have continuous first and second derivatives everywhere in $\mathbb{R}^2$ and let $R = \{x^2 + y^2 < 1, x > 0, y > 0\}$.
   (a) (10 pts) State Green's Theorem in both forms for $\boldsymbol{F}$ on R.
   (b) (16 pts) Take $\boldsymbol{F} = (y, x)$ and verify both forms on R. That is, evaluate both line integral and double integral and check that they are the same. Do this for both forms.

4. (10 pts) True or false? Give details.
   If $f(x, y, z)$ has continuous first derivatives in a domain D, and $C = \{(x(t), y(t), z(t)),\ 0 \le t \le 1\}$ be a smooth curve in D. Then $\int_C \nabla f \cdot \boldsymbol{T} ds$ depends only on f, $(x(0), y(0), z(0))$ and $(x(1), y(1), z(1))$.

5. (24 pts) Let $R = \{1/4 \le x^2 + y^2 \le 4\}$, $\boldsymbol{F}(x, y) = (2y, x)$, $\boldsymbol{G}(x, y) = (x, y)$, $\boldsymbol{H}(x, y) = (\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2})$.
   Which one(s) of $\boldsymbol{F}$, $\boldsymbol{G}$ and $\boldsymbol{H}$ is (are) conservative on R? (That is, which one(s) of $\int_C \boldsymbol{F}\cdot d\boldsymbol{r}$, $\int_C \boldsymbol{G}\cdot d\boldsymbol{r}$ and $\int_C \boldsymbol{H}\cdot d\boldsymbol{r}$ is (are) zero on <u>every</u> closed loop C in R?) Explain.

6. (15 pts) Let $\boldsymbol{F}(x, y, z) = z\boldsymbol{k}$ and $S = \{x^2 + y^2 + z^2 = 4,\ x > 0,\ y > 0,\ z > 0\}$. Evaluate $\displaystyle\int\int_S \boldsymbol{F}\cdot\boldsymbol{n}\ d\sigma$ where $\boldsymbol{n}$ is unit normal of S pointing away from the origin.

7. (10 pts) Let $f(x, y) = 2x + 3y + 4 + \sqrt{x^2+y^2}^{\frac{3}{2}}$. Is $f(x, y)$ differentiable at $(0, 0)$? Explain.

8. (10 pts) Evaluate $\displaystyle\sum_{n=0}^{\infty}\frac{x^n}{n+1}$ on $|x| < 1$ using computational rules of power series.