# Midterm 2

May 08, 2014

Show all details.

1. Evaluate
$$\frac{d}{dy}\int_1^{2+y^2}\frac{\cos(xy)}{x}dx$$

2. True or false? Prove it or give a counter example.

   Assume $f(x,y)$, $f_x(x,y)$ and $f_y(x,y)$ are all continuous in $R^2$. Let $C = \{(x,y),\ f(x,y) = f(0,0)\}$ and $\mathcal{T}$ be a tangent vector of $C$ at $(0,0)$. Then $\nabla f(0,0)\cdot\mathcal{T} = 0$.

3. Use Lagrangian multipliers (and only Lagrangian multipliers) to find extreme values of $f(x,y,z) = xy + 2z^2$ on
$$\begin{cases} x^2+y^2+z^2 = 9 \\ x - y = 0 \end{cases}$$

4. Find the absolute maximum and minimum of $f(x,y) = 2 + 2x + 2y - x^2 - y^2$ in the region bounded by $x = 0$, $y = 0$ and $x + y = 6$.

5. Let $f(x,y) = x^3 + y^3$ and $g(r,\theta) = f(r\cos\theta, r\sin\theta)$. Evaluate $\partial_r^2 g + (\partial_r g)/r + (\partial_\theta^2 g)/r^2$

6. Derive the Taylor expansion of $f(x,y,z)$ around $(0,0,0)$ up to quadratic terms of $x$, $y$ and $z$ and an expression of the remainder term, $R_2$. You may assume that $f$ and all its first and second derivatives are all continuous in $R^3$.

7. Evaluate $(\frac{\partial U}{\partial P})_V$ and $(\frac{\partial U}{\partial T})_V$ at $(P,V,T) = (1,2,2)$ where $U(P,V,T) = T\exp(-P/V)$ with the constraint $PV = T$.

8. Evaluate $\displaystyle\int_0^2\int_y^2 \frac{\sin x}{x}dxdy$.

9. Let $f(x,y) = \dfrac{x^3+y^3}{x^2+y^2}$, for $(x,y) \neq (0,0)$ and $f(0,0) = 0$. $P = (0,0)$ and $\boldsymbol{u}^\theta = (\cos\theta, \sin\theta)$, $\theta \in [0, 2\pi]$.

   (a) Is $f$ continuous at $(0,0)$? Explain.

   (b) For fixed $\theta$, write down the definition of the directional derivative $\left(\dfrac{df}{ds}\right)_{\boldsymbol{u}^\theta,P}$ and evaluate it.

   (c) Does $f$ have a linear approximation at $(0,0)$? Explain.