Midterm 1

April 01, 2014

Show all details.

1. (12 pts) Find the solution of $3xy' - y = 1 + \ln x$ on $x > 0$ with $y(1) = -2$.

2. (16 pts) Show that $\sum_{k=1}^{\infty} k^{-2}$ converges and evaluate $\lim_{n\to\infty} \dfrac{\log\left(\sum_{k=n}^{\infty} k^{-2}\right)}{\log n}$. Give details.

   Hint: If the limit is p, this means that the sum in the limit is approximately n^p. Find p and prove it.

3. (10+10 pts)

   (a) Show that the series $1 - \frac{\pi^2}{4\cdot 2!} + \frac{\pi^4}{16\cdot 4!} - \cdots + (-1)^n \frac{\pi^{2n}}{2^{2n}\cdot(2n)!} + \cdots$ converges *absolutely*. Explain.

   (b) Find the sum of the series in (a). <u>Prove your answer</u>.

   Hint: it is related to the Taylor series of an elementary function.

4. (16 pts) Give an approximation of $\int_0^{\frac{1}{2}} \sin(x^2)dx$ to within 10^{-8}. Give the formula of the approximation, need not give the numerical value. Explain why the error is less than 10^{-8}.

5. (10+10 pts) True or False? Prove it if true, give a counter example if false.

   (a) If $f(x) = a_0 + a_1x + a_2x^2 + \cdots$ on $|x| < 1$, then $a_n = f^{(n)}(0)/n!$.

   (b) If $g(x) = f(0) + \sum_{n=1}^{\infty} \frac{f^{(n)}(0)}{n!}x^n$ on $|x| < 1$, then $f(x) = g(x)$ on $|x| < 1$.

6. (16 pts) Let the region R be given as 'inside of $r = 1$' and 'outside of $r = 1 - \cos\theta$'. Find the area of R, the perimeter of R and surface area obtained from evolving R around the x-axis. Be careful about graphing R as it affects all that follows There are certain symmetry in the graph of R that you should notice.