

## Midterm Exam 2

Dec 10, 2013, 10:10AM

1. (16 pts) Graph the function  $y = \frac{x^3 + 2x - 2}{x + 1}$ . Indicate all critical points and points of inflection.

2. (10 pts) Let  $f$  be a real valued function defined on  $\{x \geq 0\}$  satisfying

(a):  $f(0) = -1$ ,

(b):  $f'(x) \geq 1/2$  for all  $x \geq 0$ .

Prove that  $f(x) = 0$  has exactly one solution on  $\{x \geq 0\}$ .

3. (16 pts) Find the limits of the following expressions:

(a)  $\lim_{x \rightarrow 0^+} x^x$       (b)  $\lim_{x \rightarrow 0} \frac{x^2 \cos \frac{1}{x}}{\sin x}$

4. (16 pts) State both parts of Fundamental Theorem of Calculus, prove that part 1 implies part 2, then evaluate

$$\frac{d}{dx} \int_{\sin x}^1 e^{t^2} dt.$$

5. (16 pts) Evaluate

(a)  $\int_1^2 \frac{1}{x(1 + \ln^2 x)} dx$       (b)  $\int_0^4 x\sqrt{2x+1} dx$

6. (10 pts) Evaluate

$$\lim_{n \rightarrow \infty} \sum_{k=n}^{2n} \frac{n}{k^2}$$

7. (16 pts) Find the volume and surface area of the object obtained by rotating the region  $\{(x-2)^2 + y^2 \leq 1, x \geq 2\}$  around the  $y$  axis. Note the surface area consists of two parts, one generated by a half circle, the other generated by a line segment.