Midterm Exam 2

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1. True or False? If true, prove it. If false, give a counter example.
   If $|f(x) - (3x+2)| \le |x|^{1.5}$ for all $x \in R$, then f is differentiable at $x = 0$.

2. (a) Graph $f(x) = \dfrac{x}{\sqrt{x^2+1}}$. Give all details including possible asymptotes.
   (b) The function $y = f(x)$ is odd ($f(-x) = -f(x)$) and the root x^* to the equation $f(x) = 0$ is $x^* = 0$. Give formula of Newton's method for finding this root.
   (c) The Newton's method does not always converge. There is an $a > 0$ such that Newton's method converges if and only if $-a < x_0 < a$. Take this fact for granted and find a (show how to find a, but need NOT prove that Newton's method converge if and only if $-a < x_0 < a$).

3. Let f be a differentiable function defined on $\{x \ge 0\}$ satisfying
   (a): $f(0) = -1$,
   (b): $f'(x) \ge 1/2$ for all $x \ge 0$.
   Show that $f(x) = 0$ has <u>one and only one</u> solution on $\{x \ge 0\}$.

4. Find the limits of the following expressions:
   $$\text{(a)} \quad \lim_{x\to 0^+} x^x \qquad \text{(b)} \quad \lim_{x\to 0^+} \frac{e^{\frac{-1}{x}}}{x} \qquad \text{(c)} \quad \lim_{x\to 0} \frac{x^2 \cos\frac{1}{x}}{\sin x}$$

5. Solve for $y(x)$ on $x < 0$ from
   $$y''(x) = x^{-2}, \quad y(-1) = 1, \quad y'(-1) = 2.$$

6. Evaluate $\displaystyle\lim_{n\to\infty} \sum_{k=n}^{2n} \frac{n}{k^2}$

7. State both parts of Fundamental Theorem of Calculus, prove that part 1 implies part 2.

8. Evaluate
   $$\text{(a)} \quad \int_1^2 \frac{1}{x(1+\ln^2 x)}dx \qquad \text{(b)} \quad \int_0^4 x\sqrt{2x+1}\,dx$$

9. True or False? If true, prove it. If false, give a counter example.
   (a) If $y = f(x)$ is differentiable at $x = c$ then it is continuous at $x = c$.
   (b) If $y = f(x)$ is continuous at $x = c$ then it is differentiable at $x = c$.

10. Start with domain and range for csc and $\csc^{-1}$, derive the formula for the derivative of $\csc^{-1}$.