Final Exam

Jan 12, 2016, 10:10AM

1. (12 pts) Find the solutions for

   (a) $\dfrac{dy}{dx} = 2x\sqrt{1-y^2}, \quad -1 < y < 1$

   (b) $\dfrac{dy}{dx} - y = -y^2, \quad$ Hint: let $u = y^{-1}$

2. (12 pts) Find the volume and surface area of the object obtained by rotating $\{(x-3)^2 + y^2 \le 1,\ x \ge 3\}$ around the y axis. Note the surface area consists of two parts, one generated by a half circle, the other generated by a line segment.

3. (10 pts) Order e^x, x^x, $(\ln x)^x$ and x^e from slowest to fastest growing rate as $x \to \infty$. Explain.

4. (6 pts) Write down the form of partial fraction expansion for $\dfrac{x^7}{(1-x^4)^2}$. Need NOT find the undetermined coefficients.

5. (50 pts)

$$(1)\ \int_1^2 \frac{x}{\sqrt{2-x}}dx \quad (2)\ \int_1^{e^\pi} \sin(\ln x)\,dx \quad (3)\ \int_0^{\pi/4} \tan^3 x \sec^3 x\,dx$$

$$(4)\ \int_1^2 \frac{1}{e^{2x}-e^{-x}}dx \quad (5)\ \int \frac{1}{\sqrt{4x-x^2}}\,dx$$

6. (10 pts) Express $\displaystyle\int_0^\pi \cos^6 x\ \sin^4 x\,dx$ in terms of $\displaystyle\int_0^\pi \cos^4 x\ \sin^4 x\,dx$.

7. (10 pts) Start with domain and range for sech and sech^{-1}, derive the formula for the derivative of sech^{-1}.

8. (8 pts) For what values of $p \in R$, is the function $f(x) = |x|^p$ differentiable at $x = 0$? Explain.

9. (12 pts) State both parts of Fundamental Theorem of Calculus, prove that part 1 implies part 2.