Midterm Exam 1

Oct 29, 2012, 10:10AM

1. (8 pts)

   Find $\lim\limits_{y \to +\infty} y \sin \dfrac{2}{\sqrt{y}}$.

2. (12 pts)

   Give precise definition of $\lim_{x \to 0^+} \frac{1}{x} = \infty$ and show that it is true (using the $\varepsilon - \delta$ argument).

3. (8 pts)

   Find dy/dx where $y = x^x, x > 0$. Need not simplify your expression.

4. (12 pts)

   Find y' and y'' at $(1, -1)$ where $y(x)$ is implicitly given by $\tan(x+y) + \sin(x^2 + y) = 0$.

5. (12 pts)

   Find the smallest n such that $\dfrac{d^n}{dx^n}(x^{10} \sin x)|_{x=0}$ is nonzero and find this value.

6. (12 pts)

   True or False? If true, prove it. If false, give a counter example.

   If $|f(x) - (3x + 2)| \le |x|^{1.5}$ for all $x \in R$, then f is differentiable at $x = 0$.

7. (12 pts)

   Write down $L(x, x_0)$, the linear approximation of f near x_0. Find an approximate value of $\sin(\frac{\pi}{3} - 0.01)$ such that the error of the approximation is smaller than 5×10^{-5}. (Hint: choose x_0 carefully)

8. (12 pts)

   True or False? (prove it if true, correct it if false).

   Since $x \mapsto \ln x$ and $x \mapsto e^x$ are inverse function to each other and $\frac{d}{dx} \ln x = \frac{1}{x}$. Therefore $\frac{d}{dx} e^x = \frac{1}{\frac{d}{dx} \ln x} = \frac{1}{\frac{1}{x}} = x$.

9. (12 pts)

   Find absolute maximum and absolute minimum of $f(x) = x^{1/3}(x - 1/2)$ on $[-1, 1]$.