Final Exam

Jan 14, 2014

1. (8 pts) Find the solutions for $\dfrac{dy}{dx} = 3x^2 e^{-y}$.

2. (8 pts) Write down the form of partial fraction expansion for $\dfrac{x^7}{(1-x^4)^2}$. Need NOT find the undetermined coefficients.

3. (8 pts) Order e^x, x^x, $(\ln x)^x$ and x^e from slowest to fastest growing rate as $x \to \infty$.

4. (64 pts) Evaluate

   (1) $\displaystyle\int \frac{1}{2+\sin x} dx$ (2) $\displaystyle\int e^x \sin x \, dx$ (3) $\displaystyle\int \frac{1}{\sqrt{4x-x^2}} \, dx$ (4) $\displaystyle\int_0^{\pi/4} \tan^3 x \sec^3 x \, dx$

   (5) $\displaystyle\int_1^2 \frac{1}{e^x - e^{-x}} dx$ (6) $\displaystyle\int_0^\infty x^2 e^{-x} dx$ (7) $\displaystyle\int_0^1 \frac{1}{\sqrt{1+e^x}} \, dx$ (8) $\displaystyle\int_0^\pi \sqrt{1+\sin x} \, dx$

5. (12 pts) Does the improper integral $\displaystyle\int_1^\infty \frac{1}{\sqrt{x^3-x}} dx$ converge or diverge? Explain (DO NOT try to integrate explicitly).

6. (8 pts) Express $\int_1^2 \sin x \, dx$ as a Riemann sum. That is, $\lim_{\cdots} \sum_{\cdots}^{\cdots} \cdots$.

7. (8 pts) Evaluate $\lim_{x \to 0^+} x^{x \ln x}$.

8. (16 pts) Find the volume and surface area of the object obtained by rotating the region $\{(x-2)^2 + y^2 \le 1, \ x \ge 2\}$ around the y axis. Note the surface area consists of two parts, one generated by a half circle, the other generated by a line segment.

9. (.100 pts) Schedule recitation change for next semester.