Midterm Exam 2

Dec 03, 2014, 10:10AM

1. (10 pts) Solve for $y(x)$ on $x < 0$ from

$$y''(x) = x^{-2}, \quad y(-1) = 1, \quad y'(-1) = 2$$

2. (12 pts) Graph $f(x) = x^{1/3}(x-4)$. Indicate all critical points and points of inflection.

3. (16 pts) Find the limits of the following expressions:

$$\text{(a)} \quad \lim_{x\to 0^+} x^x \qquad \text{(b)} \quad \lim_{x\to 0} \frac{x^2 \cos\frac{1}{x}}{\sin x}$$

4. (16 pts) Evaluate

$$\text{(a)} \quad \int_1^2 \frac{1}{x(1+\ln^2 x)} dx \qquad \text{(b)} \quad \int_0^4 x\sqrt{2x+1}\, dx$$

5. (10 pts) Evaluate

$$\lim_{n\to\infty} \sum_{k=n}^{2n} \frac{n}{k^2}$$

6. (12 pts) Let f be a real valued function defined on $\{x \geq 0\}$ satisfying

   (a): $f(0) = -1$,

   (b): $f'(x) \geq 1/2$ for all $x \geq 0$.

   Prove that $f(x) = 0$ has <u>one and only one</u> solution on $\{x \geq 0\}$.

7. (12 pts) State both parts of Fundamental Theorem of Calculus, prove that part 1 implies part 2.

8. (12 pts) Suppose $f(x)$ satisfies $\int_0^{x^2} e^{-t} f(t) dt = x$ for all $x \geq 0$. Find $f(4)$ and $f'(4)$.