

## Midterm Exam 1

Oct 23, 2014, 10:10AM

1. (10 pts)

Find  $\lim_{y \rightarrow +\infty} y \sin \frac{2}{\sqrt{y}}$ .

2. (12 pts)

Give formal definition of  $\lim_{x \rightarrow 0^+} \frac{1}{x} = \infty$  verify it using the  $\varepsilon - \delta$  argument.

3. (12 pts)

Find  $dy/dx$  where  $y = x^{(x^x)}$ ,  $x > 0$ . Need not simplify your expression.

4. (10 pts)

Find  $y'$  and  $y''$  at  $(1, -1)$  where  $y(x)$  is implicitly given by  $\tan^{-1}(x+y) + \sin^{-1}(x^2+y) = 0$  near  $(1, -1)$ .

5. (12 pts)

Find the smallest  $n$  such that  $\frac{d^n}{dx^n}(x^{10} \sin x)|_{x=0}$  is nonzero and find this value.

6. (16 pts)

True or False? If true, prove it. If false, give a counter example.

If  $|f(x) - (3x + 2)| \leq |x|^{1.5}$  for all  $x \in R$ , then  $f$  is differentiable at  $x = 0$ .

7. (12 pts)

Write down  $L(x, x_0)$ , the linear approximation of  $f$  near  $x_0$ . Find an approximate value of  $\sin(\frac{\pi}{3} - 0.01)$  such that the error of the approximation is smaller than  $5 \times 10^{-5}$ . (Hint: choose  $x_0$  carefully)

8. (16 pts)

Let  $f^{-1}$  be the inverse function of  $f$ . Evaluate  $\frac{d^2}{dy^2} f^{-1}(y)$  in terms of  $f'$  and  $f''$ . Show all details.