Final Exam

Jan 08, 2015

1. (14 pts) Find the solutions for $\dfrac{dy}{dx} = e^{x-y}$ and $x\dfrac{dy}{dx} + y = \sin x$, respectively.

2. (12 pts) Find the volume and surface area of the object obtained by rotating the region $\{(x-2)^2 + y^2 \le 1, \ x \ge 2\}$ around the y axis. Note the surface area consists of two parts, one generated by a half circle, the other generated by a line segment.

3. (8 pts) Order e^x, x^x, $(\ln x)^x$ and x^e from slowest to fastest growing rate as $x \to \infty$. Explain.

4. (6 pts) Write down the form of partial fraction expansion for $\dfrac{x^7}{(1-x^4)^2}$. Need NOT find the undetermined coefficients.

5. (64 pts) Evaluate

   (1) $\displaystyle \int \frac{1}{2+\sin x}\,dx$ (2) $\displaystyle \int_1^{e^{\pi}} \sin(\ln x)\,dx$ (3) $\displaystyle \int \frac{1}{\sqrt{4x-x^2}}\,dx$ (4) $\displaystyle \int_1^2 \frac{1}{e^{2x}-e^{-x}}\,dx$

   (5) $\displaystyle \int_0^{\pi/4} \tan^3 x \sec^3 x \, dx$ (6) $\displaystyle \int x^2 e^{-x}\,dx$ (7) $\displaystyle \int_0^1 \mathrm{sech}\,x \, dx$ (8) $\displaystyle \int_0^1 \frac{1}{\sqrt{1+e^x}}\,dx$