1. $r=f(\theta) \Rightarrow \begin{cases} x = r\cos\theta = f(\theta)\cos\theta \\ y = r\sin\theta = f(\theta)\sin\theta \end{cases}$

$$\Rightarrow \begin{cases} \frac{dx}{d\theta} = f'(\theta)\cos\theta - f(\theta)\sin\theta \\ \frac{dy}{d\theta} = f'(\theta)\sin\theta + f(\theta)\cos\theta \end{cases}$$

$$\Rightarrow \left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = (f'(\theta))^2 + (f(\theta))^2$$

$$\Rightarrow L = \int_a^b \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta = \int_a^b \sqrt{(f'(\theta))^2 + (f(\theta))^2}\, d\theta$$

2. $(\rho, \phi, \theta) = (1, \frac{\pi}{2}, \frac{\pi}{3}) \Rightarrow x = \rho\sin\phi\cos\theta = \frac{1}{2} \Rightarrow (x, y, z) = (\frac{1}{2}, \frac{\sqrt{3}}{2}, 0)$

$y = \rho\sin\phi\sin\theta = \frac{\sqrt{3}}{2}$

$z = \rho\cos\phi = 0$

$$\Rightarrow r = \sqrt{x^2+y^2} = 1 \Rightarrow (r, \theta, z) = (1, \frac{\pi}{3}, 0)$$

3. $\because g(t)$ is continuous at $t=0$, $\therefore \forall \epsilon > 0, \exists \delta_1 > 0$ s.t. if $|t-0| < \delta_1 \Rightarrow |g(t) - g(0)| < \epsilon$

$\therefore$ $\forall \epsilon > 0$, take $\delta = \delta_1 > 0$, then if $\sqrt{(x-0)^2 + (y-0)^2} < \delta \Rightarrow |t| = \sqrt{x^2+y^2} < \delta = \delta_1 \Rightarrow |g(t) - g(0)| < \epsilon$

$\Rightarrow |f(x,y) - f(0,0)| < \epsilon \Rightarrow f(x,y)$ is continuous at $(0,0)$

4. $f(x,y) = x + 2y + x^2 + y^2 \Rightarrow \begin{cases} f_x(x,y) = 1 + 2x \\ f_y(x,y) = 2 + 2y \end{cases} \Rightarrow \begin{cases} f_x(0,0) = 1 \\ f_y(0,0) = 2 \end{cases}$

$$\Rightarrow L(x,y) = f(0,0) + f_x(0,0)(x-0) + f_y(0,0)(y-0)$$
$$= x + 2y$$

$$\Rightarrow a=1,\ b=2,\ c=0$$

$$\Rightarrow \frac{f(x,y)-L(x,y)}{\sqrt{x^2+y^2}} = \frac{x^2+y^2}{\sqrt{x^2+y^2}} = \sqrt{x^2+y^2}$$

Show that $\lim_{(x,y)\to(0,0)} \sqrt{x^2+y^2} = 0$

$\forall \epsilon > 0$, take $\delta = \epsilon$, then if $\sqrt{(x-0)^2+(y-0)^2} < \delta$, $\left|\sqrt{x^2+y^2} - 0\right| < \delta = \epsilon$

$$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{f(x,y)-L(x,y)}{\sqrt{x^2+y^2}} = \lim_{(x,y)\to(0,0)} \sqrt{x^2+y^2} = 0$$

5. Let $a(y) = 2+\sin y$, $z(y) = y$

$$\Rightarrow \int_1^{2+\sin y} \frac{\cos(xy)}{x}\,dx = \int_1^{a(y)} \frac{\cos(x z(y))}{x}\,dx \overset{\text{let}}{=} F(a(y), z(y))$$

$$\Rightarrow \frac{d}{dy}\int_1^{2+\sin y} \frac{\cos(xy)}{x}\,dx = \frac{d}{dy} F(a(y), z(y)) = \frac{\partial F(a,z)}{\partial a}\cdot\frac{da}{dy} + \frac{\partial F(a,z)}{\partial z}\cdot\frac{dz}{dy}$$

$$= \frac{\cos(az)}{a}\cdot \cos y + \int_1^a -\sin(xz)\,dx \cdot 1$$

$$= \frac{\cos((2+\sin y)y)\cos y}{2+\sin y} + \frac{1}{z}\cos(xz)\Big|_{x=1}^{x=a}$$

$$= \frac{\cos((2+\sin y)y)\cos y}{2+\sin y} + \frac{1}{z}\left(\cos(az) - \cos z\right)$$

$$= \frac{\cos((2+\sin y)y)\cos y}{2+\sin y} + \frac{1}{y}\left(\cos((2+\sin y)y) - \cos y\right)$$